BOARD OF STUDIES
NEW SOUTH WALES

Mathematics
Years 7–10

Syllabus

(incorporating Content and Outcomes for Stage 2 to Stage 5)

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1 Introduction

1.1 The K–10 Curriculum

This syllabus has been developed within the parameters set by the Board of Studies NSW in its *K–10 Curriculum Framework*. This framework ensures that K–10 syllabuses and curriculum requirements are designed to provide educational opportunities that:

- engage and challenge all students to maximise their individual talents and capabilities for lifelong learning
- enable all students to develop positive self-concepts, and their capacity to establish and maintain safe, healthy and rewarding lives
- prepare all students for effective and responsible participation in their society, taking account of moral, ethical and spiritual considerations
- encourage and enable all students to enjoy learning, and to be self-motivated, reflective, competent learners who will be able to take part in further study, work or training
- promote a fair and just society that values diversity
- promote continuity and coherence of learning and facilitate transition between primary and secondary schooling.

The framework also provides a set of broad learning outcomes that summarise the knowledge, skills and understanding, values and attitudes essential for all students to succeed in and beyond their schooling. These broad learning outcomes indicate that students will:

- understand, develop and communicate ideas and information
- access, analyse, evaluate and use information from a variety of sources
- work collaboratively with others to achieve individual and collective goals
- possess the knowledge and skills necessary to maintain a safe and healthy lifestyle
- understand and appreciate the physical, biological and technological world and make responsible and informed decisions in relation to their world
- understand and appreciate social, cultural, geographical and historical contexts and participate as active and informed citizens
- express themselves through creative activity and engage with the artistic, cultural and intellectual work of others
- understand and apply a variety of analytical and creative techniques to solve problems
- understand, interpret and apply concepts related to numerical and spatial patterns, structures and relationships
- be productive, creative and confident in the use of technology and understand the impact of technology on society
- understand the work environment and be equipped with the knowledge, skills and understanding to evaluate potential career options and pathways
- develop a system of personal values based on their understanding of moral, ethical and spiritual matters.

The way in which learning in the *Mathematics Years 7–10 Syllabus* contributes to curriculum and to the student's achievement of the broad learning outcomes is outlined in the syllabus rationale.

In accordance with the *K–10 Curriculum Framework*, the *Mathematics Years 7–10 Syllabus* takes into account the diverse needs of all students. It identifies essential knowledge, skills and understanding, values and attitudes. It enunciates clear standards of what students are expected to know and be able to do in Years 7–10. It provides structures and processes by which teachers can provide continuity of study for all students, particularly to ensure successful transition from Years 5 to 8 and from Years 10 to 11. It offers advice to teachers on ways of addressing the needs of students in Years 7–10 who have not achieved Stage 2 or Stage 3 outcomes.

The syllabus also assists students to maximise their achievement in Mathematics through the acquisition of additional knowledge, skills and understanding, values and attitudes. It contains advice to assist teachers to program learning for those students who have gone beyond achieving the outcomes through their study of the essential content.

1.2 Learning in Mathematics in K–10

A K–10 Mathematics Scope and Continuum that describes the content to be developed at each Stage, and for each content strand (Number, Patterns and Algebra, Data, Measurement, Space and Geometry), has been developed to assist teachers in planning for learning in Mathematics from Kindergarten to Year 10.

The content presented in any particular Stage represents the knowledge, skills and understanding that are to be achieved by a typical student by the end of that Stage. It needs to be acknowledged that students learn at different rates and in different ways, so that there will be students who have not achieved the outcomes for the Stage/s prior to that identified with their stage of schooling. Teachers will need to identify these students and to plan learning experiences that provide opportunities for the development of understanding of earlier concepts. In addition, there will be students who achieve the outcomes for their Stage before the end of their stage of schooling. These students will need learning experiences that develop understanding of concepts in the next Stage. In this way, students can move through the continuum at a faster rate.

For example, some students will achieve Stage 4 outcomes during Year 7, while the majority will achieve them by the end of Year 8. Other students might not develop the same understanding until Year 9 or later. Consequently, three specific endpoints and pathways (5.1, 5.2 and 5.3) have been identified for Stage 5. Other endpoints and pathways are also possible in Stage 5; for example, some students may achieve all the 5.2 outcomes and a selection of some of the 5.3 outcomes.

In order to meet students' vocational and other learning needs beyond the compulsory years, a variety of mathematical learning experiences are required in Years 9 and 10. The arrangement of content in Stage 5 acknowledges the wide range of achievement of students in Mathematics as they enter the last two years of their compulsory years of schooling. Stage 5.1 content is designed to meet the needs of students who achieve Stage 4 outcomes during Year 9 or Year 10. Stage 5.2 content builds on and includes the content of Stage 5.1 and is designed for students who have achieved Stage 4 content generally by the end of Year 8 or early in Year 9. Stage 5.3 content includes the content for 5.1 and 5.2 and is designed for students who have achieved Stage 4 outcomes probably before the end of Year 8. Although the syllabus arranges the content in Stages, it is written with the flexibility to enable students to work at different Stages in different content areas. For example, students could be working on Stage 4 content for Number and Stage 3 content for Measurement.

When planning learning experiences for students in Years 9 and 10, teachers need to consider pathways that students plan to follow beyond Year 10. For students who intend to study the Stage 6 General Mathematics course, it is recommended that they experience at least some of the 5.2 content, particularly the Patterns and Algebra topics and *Trigonometry*, if not all of the content. For students who intend to study the Stage 6 Mathematics course, it is recommended that they experience the topics *Real Numbers*, *Algebraic Techniques* and *Coordinate Geometry* as well as at least some of *Trigonometry* and *Deductive Geometry* from 5.3 (identified by §), if not all of the content. For students who intend to study the Stage 6 Mathematics Extension 1 course, it is recommended that they experience the optional topics (identified by #) *Curve Sketching and Polynomials*, *Functions and Logarithms*, and *Circle Geometry*.

To achieve the outcomes for Stages 4 and 5, the use of a calculator that incorporates the features generally associated with 'scientific' calculators is mandatory. Graphics calculators include these features and it is important to be aware that graphics calculators (that conform to requirements notified by the Office of the Board of Studies) are permitted in the Office's testing of Mathematics for the School Certificate. It is important, however, that students maintain and develop their mental arithmetic skills, rather than relying on their calculators for every calculation.

1.3 Students with Special Education Needs

In the K–6 curriculum, students with special education needs are provided for in the following ways:

- through the inclusion of outcomes and content in syllabuses which provide for the full range of students
- through the development of additional advice and programming support for teachers to assist students to access the outcomes of the syllabus
- through the development of specific support documents for students with special education needs
- teachers and parents planning together to ensure that syllabus outcomes and content reflect the learning needs and priorities of students.

Students with special education needs build on their achievements in K–6 as they progress through their secondary study and undertake courses to meet the requirements for the School Certificate.

It is necessary to continue focusing on the needs, interests and abilities of each student when planning a program for secondary schooling. The program will comprise the most appropriate combination of courses, outcomes and content available.

Life Skills

For most students with special education needs, the outcomes and content in sections 7 and 9 of this Syllabus will be appropriate but for a small percentage of these students, particularly those with an intellectual disability, it may be determined that these outcomes and content are not appropriate. For these students the Life Skills outcomes and content in section 10 and the Life Skills assessment advice below can provide the basis for developing a relevant and meaningful program.

Access to Life Skills outcomes and content in Years 7–10

A decision to allow a student to access the Mathematics Years 7–10 Life Skills outcomes and content should include parents/carers and be based on careful consideration of the student's competencies and learning needs.

The decision should establish that the outcomes and content in sections 7 and 9 of this Syllabus are not appropriate to meet the needs of the student. Consideration should be given to whether modifications to programs and to teaching, including adjustments to learning activities and assessment, would enable the student to access the syllabus outcomes and content.

As part of the decision to allow a student to access the Mathematics Years 7–10 Life Skills outcomes and content, it is important to identify relevant settings, strategies and resource requirements that will assist the student in the learning process. Clear time frames and strategies for monitoring progress, relevant to the age of the student, need to be identified and collaborative plans should be made for future needs.

It is not necessary to seek permission of the Office of the Board of Studies for students to undertake the Mathematics Years 7–10 Life Skills outcomes and content, nor is it necessary to submit planning documentation.

Life Skills Assessment

Each student undertaking a Mathematics Years 7–10 Life Skills course will have specified outcomes and content to be studied. The syllabus content listed for each outcome forms the basis of learning opportunities for students.

Assessment should provide opportunities for students to demonstrate achievement in relation to the outcomes and to generalise their knowledge, understanding and skills across a range of situations or environments including the school and the wider community.

Students may demonstrate achievement in relation to Mathematics Years 7–10 Life Skills outcomes independently or with support. The type of support will vary according to the particular needs of the student and the requirements of the activity. Examples of support may include:

- the provision of extra time
- physical and/or verbal assistance from others
- the provision of technological aids.

2 Rationale for Mathematics in K–10

Mathematics is a reasoning and creative activity employing abstraction and generalisation to identify, describe and apply patterns and relationships. It is a significant part of the cultural heritage of many diverse societies. The symbolic nature of mathematics provides a powerful, precise and concise means of communication. Mathematics incorporates the processes of questioning, reflecting, reasoning and proof. It is a powerful tool for solving familiar and unfamiliar problems both within and beyond mathematics. As such, it is integral to scientific and technological advances in many fields of endeavour. In addition to its practical applications, the study of mathematics is a valuable pursuit in its own right, providing opportunities for originality, challenge and leisure.

The study of mathematics provides opportunities for students to learn to describe and apply patterns and relationships; reason, predict and solve problems; calculate accurately both mentally and in written form; estimate and measure; and interpret and communicate information presented in numerical, geometrical, graphical, statistical and algebraic forms. Mathematics in K–10 provides support for concurrent learning in other key learning areas and builds a sound foundation for further mathematics education.

Students will have the opportunity to develop an appreciation of mathematics and its applications in their everyday lives and in the worlds of science, technology, commerce, the arts and employment. The study of the subject enables students to develop a positive self-concept as learners of mathematics, obtain enjoyment from mathematics, and become self-motivated learners through inquiry and active participation in challenging and engaging experiences.

The ability to make informed decisions, and to interpret and apply mathematics in a variety of contexts, is an essential component of students' preparation for life in the twenty-first century. To participate fully in society students need to develop the capacity to critically evaluate ideas and arguments that involve mathematical concepts or that are presented in mathematical form.

3 Place of the Mathematics Years 7–10 Syllabus in the K–12 Curriculum

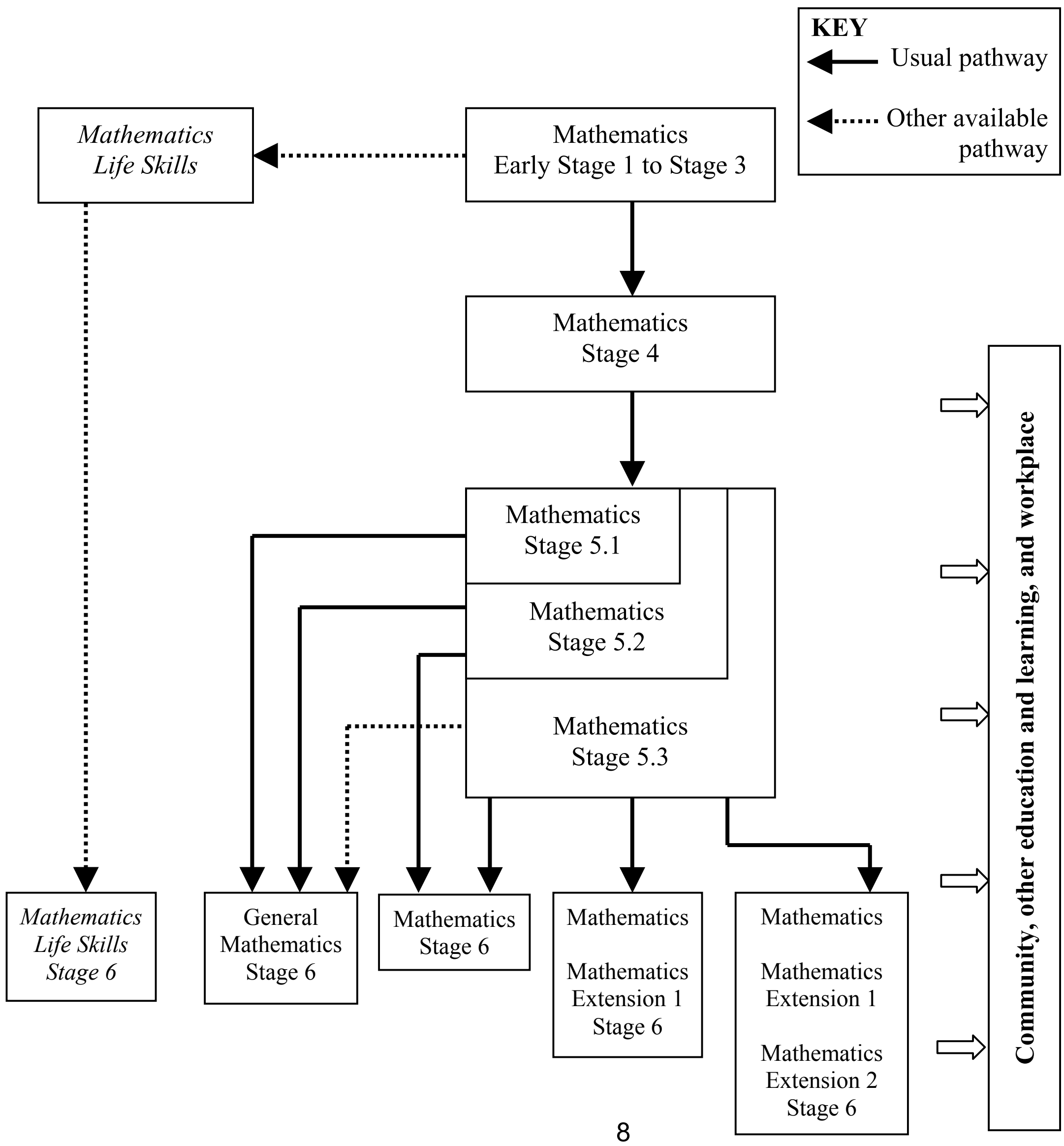
3.1 Pathways of Learning

The *Mathematics Years 7–10 Syllabus* forms part of the continuum of mathematics learning from Kindergarten to Year 10. To ensure coherence and continuity, this syllabus was developed at the same time as the *Mathematics K–6 Syllabus*. The Stage 6 syllabuses were developed for students in Years 11 and 12 and therefore represent the mathematics learning for all students who study mathematics in those years.

The following diagram represents available pathways of learning in mathematics from Early Stage 1 to Stage 6. In this diagram, the Stages refer to the level of knowledge of mathematics learning rather than to the stages of schooling. In this way it is acknowledged that a student who is in Year 7, for example, may still be working towards Stage 3 outcomes, just as a student in Year 8, for example, may be working towards Stage 5 outcomes.

The Mathematics Life Skills outcomes and content are designed to provide a relevant and meaningful program of study for a small percentage of students with special education needs, for whom the Mathematics Years 7–10 Syllabus outcomes and content are not appropriate.

In order to cater for the full range of learners, three specific endpoints and pathways (5.1, 5.2 and 5.3) have been identified for Stage 5. The diagram shows the connection between these three levels. Stage 5.3 includes the knowledge and skills from Stage 5.2, and Stage 5.2 includes the knowledge and skills from Stage 5.1.



3.2 Cross-curriculum Content

The Board of Studies has developed cross-curriculum content that is to be included in the outcomes and content of syllabuses. The identified content will be incorporated appropriately in K–10 syllabuses. The cross-curriculum content addresses issues, perspectives and policies that will assist students to achieve the broad learning outcomes defined in the Board of Studies *K–10 Curriculum Framework.* The cross-curriculum content statements have been developed in accordance with the requirement of the *K–10 Curriculum Framework* that 'syllabuses will include cross-curriculum content that is appropriate to teach in the key learning area or subject'.

The statements act as a mechanism to embed cross-curriculum content into all syllabuses for K–10. Knowledge, skills, understanding, values and attitudes derived from the cross-curriculum content areas will be included in Board syllabuses, while ensuring that subject integrity is maintained.

Information and Communication Technology (ICT) has been developed with the significant utilisation of mathematics, and a range of opportunities exists within the teaching and learning of mathematics to utilise ICT. For example, spreadsheets may contribute to activities related to tabulating and graphing data sets in both the Patterns and Algebra and Data strands. Graphics calculators can be used for many applications including exploring data sets and investigating curves. Dynamic geometry software packages can assist in illustrating and establishing the properties of geometric figures. Random number generators can facilitate simulation experiments in probability. Data loggers may be used in investigations related to graphing relationships.

Work, Employment and Enterprise content enables students to develop work-related knowledge, skills and understanding through their study of mathematics. It also provides opportunities for students to develop values and attitudes about work, employment and the workplace.

Specifically this occurs through student study of mathematics in work-related contexts, through selecting and applying appropriate mathematical techniques and problem-solving strategies, and in acquiring, processing, assessing and communicating information.

Numeracy is the ability to effectively use the mathematics required to meet the general demands of life at home and at work, and for participation in community and civic life. As a field of study, mathematics is developed and/or applied in situations that extend beyond the general demands of everyday life.

Numeracy is a fundamental component of learning across all areas of the curriculum. The development and enhancement of students' numeracy skills and understanding is the responsibility of teachers across different learning areas that make specific demands on student numeracy.

To be numerate is to use mathematical ideas effectively to make sense of the world. Numeracy involves drawing on knowledge of particular contexts and circumstances in deciding when to use mathematics, choosing the mathematics to use, and critically evaluating its use. Numeracy incorporates the disposition to use numerical, spatial, graphical, statistical and algebraic concepts and skills in a variety of contexts and involves the critical evaluation, interpretation, application and communication of mathematical information in a range of practical situations.

The key role that teachers of mathematics play in the development of numeracy includes teaching students specific skills and providing them with opportunities to select, use, evaluate and communicate mathematical ideas in a range of situations. Students' numeracy and underlying mathematical understanding will be enhanced through engagement with a variety of applications of mathematics to real-world situations and problems in other key learning areas.

Key Competencies are generic competencies essential for effective participation in existing and emerging learning for future education, work and life in general. The *Mathematics Years 7–10 Syllabus* provides a powerful context within which to develop general competencies considered essential for the continuing development of those effective thinking skills which are necessary for further education, work and everyday life. The knowledge, skills and understanding that underpin the key competencies are taught by making them explicit, designing learning tasks that provide opportunities to develop them, and identifying specific criteria for their assessment.

Key competencies are embedded in the *Mathematics Years 7–10 Syllabus* to enhance student learning. They are incorporated into the objectives, outcomes and content of the syllabus and/or are developed through classroom teaching. The key competencies are:

- collecting, analysing and organising information
- communicating ideas and information
- planning and organising activities
- working with others and in teams
- using mathematical ideas and techniques
- solving problems
- using technology.

This syllabus explicitly addresses knowledge and skills that provide students with opportunities to *collect, analyse and organise information* numerically and graphically.

Mathematics contributes to the development of students' abilities to *communicate ideas and information* by facilitating the development of skills in interpreting and representing information in numerical, algebraic, statistical and graphical forms. Students are encouraged to express mathematical concepts and processes using their own words as well as using mathematical terminology and notation.

Problem-solving tasks provide opportunities for students to develop the capacity to *plan and organise activities.* Planning and organising their own strategies for obtaining solutions to tasks involves the ability to set goals, establish priorities, implement a plan, select and manage resources and time, and monitor individual performance.

The experience of *working with others and in teams* can facilitate learning. Groupwork provides the opportunity for students to communicate mathematically with each other, to make conjectures, to cooperate and to persevere when solving problems and undertaking investigations.

Throughout the syllabus, students are developing the key competencies *using mathematical ideas and techniques* and *solving problems*. Across the syllabus strands attention is drawn to opportunities for students to solve meaningful and challenging problems in both familiar and unfamiliar contexts, within mathematics, in other key learning areas, at work and in everyday situations. Problem solving can promote communication, critical reflection, creativity, analysis, organisation, experimentation, synthesis, generalisation, validation, perseverance, and systematic recording of information. In addition, teaching through problems that are relevant to students can encourage improved attitudes to mathematics and an appreciation of its importance to society.

In order to achieve the outcomes of this syllabus, students will need to learn about and use appropriate technologies to develop the key competency *using technology.* It is important for students to determine the purpose of a technology, when and how to apply the technology, and to evaluate the effectiveness of its application, or whether its use is inappropriate or even counterproductive. Computer software as well as scientific and graphics calculators can be used to facilitate teaching and learning.

Literacy is the ability to communicate purposefully and appropriately with others, in and through a wide variety of contexts, modes and mediums. While English has a particular role in developing literacy, all curriculum areas, including mathematics, have a responsibility for the general literacy requirements of students, as well as for the literacy demands of their particular discipline.

Mathematics language is concise and precise. Students are taught mathematical vocabulary and the conventions for writing mathematics. Studies have shown that the causes of student errors on word problems may relate to the literacy components rather than the application of mathematical computations. Mathematics at times uses words from everyday language that have different meanings within a mathematical context. This can add to some students' confusion. Clear explanations of these differences will assist students in the acquisition and use of mathematical terminology.

The growth of technology and information, including visual information, demands that students be critically, visually and technologically literate and can compose, acquire, process, and evaluate text in a wide variety of contexts. They need to understand the full scope of a text's meaning, including the wide contextual factors that take meaning beyond a decoding process.

4 Aim

The aim of Mathematics in K–10 is to develop students' mathematical thinking, understanding, competence and confidence in the application of mathematics, their creativity, enjoyment and appreciation of the subject, and their engagement in lifelong learning.

5 Objectives

Knowledge, Skills and Understanding

Students will develop knowledge, skills and understanding:

- through inquiry, application of problem-solving strategies including the selection and use of appropriate technology, communication, reasoning and reflection
- in mental and written computation and numerical reasoning
- in patterning, generalisation and algebraic reasoning
- in collecting, representing, analysing and evaluating information
- in identifying and quantifying the attributes of shapes and objects and applying measurement strategies
- in spatial visualisation and geometric reasoning.

Values and Attitudes

Students will:

- appreciate mathematics as an essential and relevant part of life
- show interest and enjoyment in inquiry and the pursuit of mathematical knowledge, skills and understanding
- demonstrate confidence in applying mathematical knowledge, skills and understanding to everyday situations and the solution of everyday problems
- develop and demonstrate perseverance in undertaking mathematical challenges
- recognise that mathematics has been developed in many cultures in response to human needs.

6 Syllabus Structure

This syllabus contains essential and additional content. The essential content is presented as outcomes and content statements in six strands. The additional content consists of non-mandatory topics that teachers may use to further broaden and enrich students' learning in mathematics. As well as the essential and additional content, particular cross-curriculum areas are incorporated into the content of the syllabus as described on pages 9–10.

Essential Content

The essential content for Mathematics in K–10 is structured using one process strand

- Working Mathematically

and five content strands

- Number
- Patterns and Algebra
- Data
- Measurement
- Space and Geometry.

These strands contain the knowledge, skills and understanding for the study of mathematics in the compulsory years of schooling.

Each strand is linked to an objective.

Strand	Objective
Working Mathematically	Students will develop knowledge, skills and understanding through inquiry, application of problem-solving strategies including the selection and use of appropriate technology, communication, reasoning and reflection.
Number	Students will develop knowledge, skills and understanding in mental and written computation and numerical reasoning.
Patterns and Algebra	Students will develop knowledge, skills and understanding in patterning, generalisation and algebraic reasoning.
Data	Students will develop knowledge, skills and understanding in collecting, representing, analysing and evaluating information.
Measurement	Students will develop knowledge, skills and understanding in identifying and quantifying the attributes of shapes and objects and applying measurement strategies.
Space and Geometry	Students will develop knowledge, skills and understanding in spatial visualisation and geometric reasoning.

Strands are used as organisers of mathematical outcomes and content to assist teachers with planning, programming, assessment and reporting. From Early Stage 1 to Stage 3, the five content strands are organised into substrands, and in Stage 4 the strands are organised into topics. The following table summarises the organisational structure for Mathematics from Early Stage 1 to Stage 5.3. The substrands for Early Stage 1 to Stage 3 are the same and have been included as one column.

Some topics in the table have been identified with one of two symbols (§ and #). This has been done to assist teachers when planning learning experiences for students in Years 9 and 10 in relation to the pathways that they plan to follow beyond Year 10. For students who intend to study the Stage 6 General Mathematics course, it is recommended that they experience at least some of the 5.2 content, particularly the Patterns and Algebra topics and *Trigonometry*, if not all of the content. For students who intend to study the Stage 6 Mathematics course, it is recommended that they experience the topics *Real Numbers*, *Algebraic Techniques* and *Coordinate Geometry* as well as at least some of *Trigonometry* and *Deductive Geometry* from 5.3 (identified by §), if not all of the content. For students who intend to study the Stage 6 Mathematics Extension 1 course, it is recommended that they experience the optional topics (identified by #) *Curve Sketching and Polynomials*, *Functions and Logarithms*, and *Circle Geometry*.

			Stage 5.3		
			Stage 5.2		
Strand	**Early Stage 1 to Stage 3**	**Stage 4**	**Stage 5.1**		
Working Mathematically	**Five Interrelated Processes** Questioning Applying Strategies Communicating Reasoning Reflecting				
Number	Whole Numbers Addition and Subtraction Multiplication and Division Fractions and Decimals Chance	Operations with Whole Numbers Integers Fractions, Decimals and Percentages Probability	Rational Numbers Consumer Arithmetic Probability	Rational Numbers Consumer Arithmetic	§ Real Numbers Probability
Patterns and Algebra	Patterns and Algebra	Number Patterns Algebraic Techniques Linear Relationships	Algebraic Techniques Coordinate Geometry	Algebraic Techniques Coordinate Geometry Graphs of Physical Phenomena	§ Algebraic Techniques § Coordinate Geometry Graphs of Physical Phenomena # Curve Sketching and Polynomials # Functions and Logarithms
Data	Data	Data Representation Data Analysis and Evaluation	Data Representation and Analysis	Data Analysis and Evaluation	
Measurement	Length Area Volume and Capacity Mass Time	Perimeter and Area Surface Area and Volume Time	Perimeter and Area Trigonometry	Perimeter and Area Surface Area and Volume Trigonometry	Surface Area and Volume § Trigonometry
Space and Geometry	Three-dimensional Space Two-dimensional Space Position	Properties of Solids Angles Properties of Geometrical Figures		Properties of Geometrical Figures	§ Deductive Geometry # Circle Geometry

(# optional topics as further preparation for the Mathematics Extension courses in Stage 6)
(§ recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course)

Additional Content

In addition to the content covered by the outcomes listed in each of the strands, teachers may wish to include in their teaching and learning programs other material in order to broaden and deepen students' knowledge, skills and understanding, to meet students' interests, or to stimulate student interest in other areas of mathematics.

The following list contains possible topics for inclusion as Additional Content in teaching and learning programs. This additional content is not essential, nor is it required as prerequisite knowledge for other topics in the K–12 Mathematics curriculum. The list is not exhaustive.

Number

Exploration of numbers such as perfect and amicable numbers
Set theory
Venn diagrams
Number bases other than 10
Calculating methods and devices eg abacus, Napier's Bones
Construction of magic squares
Cube root formula
Algorithm for finding square roots
Logic puzzles
Number theory
Matrices and vectors

Patterns and Algebra

Linear programming
Finite differences
Three-dimensional coordinate geometry
Polar coordinates

Data

The normal distribution

Measurement

Heron's formula for the area of a triangle
Non-metric units of measurement
Surveying
Navigation – latitude and longitude

Space and Geometry

Codes
Knots
Further tessellations (including semi-regular tessellations)
Networks
Topology
Planes of symmetry of solids
Semi-regular polyhedra; truncated, snub-nosed and stellated solids
Construction of inscribed, circumscribed and escribed circles for a triangle
Construction, using ruler and compasses, of angles of 15º, 30º, 45º, 60º, 90º, 105º
Fractals
Golden section
Golden mean construction

Indicative Hours

In accordance with the requirements of the *K–10 Curriculum Framework*, this syllabus has been designed so that students would typically achieve the standards described in 400 hours. These indicative hours will provide the basis for timetabling and programming decisions. Some students may achieve a particular standard in fewer hours while others may require additional time to achieve the standard. Students entering Year 7 who have not achieved Stage 3 outcomes may need additional time.

7 Outcomes for Early Stage 1 to Stage 5

The syllabus outcomes are arranged in Stages that follow a conceptual sequence from Early Stage 1 to Stage 5. The outcomes and content for each strand and substrand describe the knowledge, skills and understanding to be developed by most students at each Stage. However, it is acknowledged that students learn at different rates and in different ways. In order to cater for the full range of students in Years 7–10, Stages 2 and 3 outcomes and content have been included in this syllabus. In addition, it is possible that there will be variability in the achievement of Stage outcomes at particular Years of schooling. For example, some students will achieve the Stage 4 outcomes during Year 7, while the majority will achieve them by the end of Year 8. Other students might not achieve them until Year 9 or later. Consequently, three specific endpoints and pathways (5.1, 5.2 and 5.3) have been identified for Stage 5. Other endpoints and pathways are also possible in Stage 5. For example, some students may achieve all of the 5.2 outcomes and a selection of some of the 5.3 outcomes.

A code has been applied to each of the outcomes to facilitate reference throughout the syllabus.

WM	Working Mathematically
N	Number
PA	Patterns and Algebra
D	Data
M	Measurement
SG	Space and Geometry

For example, the following outcome:

NS4.3 Operates with fractions, decimals, percentages, ratios and rates

refers to an outcome from the Number strand in Stage 4. The last number indicates that this outcome belongs to the third set of Number outcomes.

In the Stages from Early Stage 1 to Stage 3, where there is more than one outcome for a substrand at a particular Stage, the code ends with ‘a’ or ‘b’ to indicate the first or second outcome.

For example, the following two outcomes are included in Two-dimensional Space for Stage 3:

SGS3.2a Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties

SGS3.2b Measures, constructs and classifies angles

Working Mathematically Outcomes

There is no specific list of knowledge and skills for the Working Mathematically strand. The Working Mathematically processes have been embedded in the content section of this syllabus and appear on each of the content pages. The wording of the outcomes for Questioning and Reflecting is the same for each Stage except for the last part of the statement, which indicates that the outcome should be assessed in relation to the relevant content for that Stage. This is not to suggest that there is no development of these two processes across Stages. Development of these processes is closely linked to the development of the content and needs to be assessed in relation to the content. For Questioning, this means that a student working towards Stage 3 might ask a question about creating sixths of a collection of objects whereas a student working towards Stage 5.2 might ask a question about the impact of adding a constant term to the graph of a simple parabola. For Reflecting, a student working towards Stage 3 might be able to make connections between areas of rectangles and multiplication whereas a student working towards Stage 4 might be able to make connections between finding areas of triangles and using Pythagoras’ theorem to find the length of an unknown perpendicular height.

The following tables list the outcomes (with page reference to the relevant content contained in this syllabus) for each strand and for each Stage from Early Stage 1 to Stage 5. The full range of outcomes is presented in this syllabus so that teachers are aware of the progression across the Stages. The table shows the connection between three levels in Stage 5. Stage 5.3 includes the outcomes from Stage 5.1 and Stage 5.2. Stage 5.2 includes the outcomes from Stage 5.1. Topics in 5.3 identified by a # are optional topics to enable further preparation for the Mathematics Extension courses in Stage 6. Topics in 5.3 marked with a § are recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course.

Working Mathematically Outcomes for Early Stage 1 to Stage 3

Process	Early Stage 1	Stage 1	Stage 2	Stage 3
Questioning Students ask questions in relation to mathematical situations and their mathematical experiences	WMES1.1 Asks questions that could be explored using mathematics in relation to Early Stage 1 content	WMS1.1 Asks questions that could be explored using mathematics in relation to Stage 1 content	WMS2.1 Asks questions that could be explored using mathematics in relation to Stage 2 content	WMS3.1 Asks questions that could be explored using mathematics in relation to Stage 3 content
Applying Strategies Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and solve problems	WMES1.2 Uses objects, actions, imagery, technology and/or trial and error to explore mathematical problems	WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems	WMS2.2 Selects and uses appropriate mental or written strategies, or technology, to solve problems	WMS3.2 Selects and applies appropriate problem-solving strategies, including technological applications, in undertaking investigations
Communicating Students develop and use appropriate language and representations to formulate and express mathematical ideas	WMES1.3 Describes mathematical situations using everyday language, actions, materials and informal recordings	WMS1.3 Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols	WMS2.3 Uses appropriate terminology to describe, and symbols to represent, mathematical ideas	WMS3.3 Describes and represents a mathematical situation in a variety of ways using mathematical terminology and some conventions
Reasoning Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions	WMES1.4 Uses concrete materials and/or pictorial representations to support conclusions	WMS1.4 Supports conclusions by explaining or demonstrating how answers were obtained	WMS2.4 Checks the accuracy of a statement and explains the reasoning used	WMS3.4 Gives a valid reason for supporting one possible solution over another
Reflecting Students reflect on their experiences and critical understanding to make connections with, and generalisations about, existing knowledge and understanding	WMES1.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Early Stage 1 content	WMS1.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 1 content	WMS2.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 2 content	WMS3.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 3 content

Working Mathematically Outcomes for Stage 4 to Stage 5

Process	Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Questioning Students ask questions in relation to mathematical situations and their mathematical experiences	WMS4.1 Asks questions that could be explored using mathematics in relation to Stage 4 content	WMS5.1.1 Asks questions that could be explored using mathematics in relation to Stage 5.1 content	WMS5.2.1 Asks questions that could be explored using mathematics in relation to Stage 5.2 content	WMS5.3.1 Asks questions that could be explored using mathematics in relation to Stage 5.3 content
Applying Strategies Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and develop solutions in solving problems	WMS4.2 Analyses a mathematical or real-life situation, solving problems using technology where appropriate	WMS5.1.2 Analyses a mathematical or real-life situation, systematically applying problem-solving strategies	WMS5.2.2 Selects and uses appropriate problem-solving strategies that include selecting and organising key information and identifying and working on related problems	WMS5.3.2 Solves problems using a range of strategies including deductive reasoning
Communicating Students develop and use appropriate language and representations to formulate and express mathematical ideas	WMS4.3 Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas	WMS5.1.3 Uses mathematical terminology and notation, algebraic symbols, diagrams, text and tables to explain mathematical ideas	WMS5.2.3 Uses appropriate mathematical language and algebraic, statistical and other notations and conventions in written, oral or graphical form	WMS5.3.3 Uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures
Reasoning Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions	WMS4.4 Identifies relationships and the strengths and weaknesses of different strategies and solutions, giving reasons	WMS5.1.4 Explains and verifies mathematical relationships	WMS5.2.4 Uses mathematical arguments to reach and justify conclusions	WMS5.3.4 Uses deductive reasoning in presenting arguments and formal proofs
Reflecting Students reflect on their experiences and critical understanding to make connections with, and generalisations about, existing knowledge and understanding	WMS4.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 4 content	WMS5.1.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 5.1 content	WMS5.2.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 5.2 content	WMS5.3.5 Links mathematical ideas and makes connections with, and generalisations about, existing knowledge and understanding in relation to Stage 5.3 content

Number Outcomes for Early Stage 1 to Stage 3

Substrand	Early Stage 1	Stage 1	Stage 2	Stage 3
Whole Numbers Students develop *a sense* of the relative size of whole numbers and the role of place value in their representation	NES1.1 Counts to 30, and orders, reads and represents numbers in the range 0 to 20	NS1.1 Counts, orders, reads and represents two- and three-digit numbers	NS2.1 (p 48) Counts, orders, reads and records numbers up to four digits	NS3.1 (p 49) Orders, reads and writes numbers of any size
Addition and Subtraction Students develop facility with number facts and computation with progressively larger numbers in addition and subtraction and an appreciation of the relationship between those facts	NES1.2 Combines, separates and compares collections of objects, describes using everyday language and records using informal methods	NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers	NS2.2 (p 50) Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers	NS3.2 (p 52) Selects and applies appropriate strategies for addition and subtraction with counting numbers of any size
Multiplication and Division Students develop facility with number facts and computation with progressively larger numbers in multiplication and division and an appreciation of the relationship between those facts	NES1.3 Groups, shares and counts collections of objects, describes using everyday language and records using informal methods	NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division	NS2.3 (p 53) Uses mental and informal written strategies for multiplication and division	NS3.3 (p 55) Selects and applies appropriate strategies for multiplication and division
Fractions and Decimals Students develop an understanding of the parts of a whole, and the relationships between the different representations of fractions	NES1.4 Describes halves, encountered in everyday contexts, as two equal parts of an object	NS1.4 Describes and models halves and quarters, of objects and collections, occurring in everyday situations	NS2.4 (p 59) Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages	NS3.4 (p 61) Compares, orders and calculates with decimals, simple fractions and simple percentages
Chance Students develop an understanding of the application of chance in everyday situations and an appreciation of the difference between theoretical and experimental probabilities	No outcome at this Stage	NS1.5 Recognises and describes the element of chance in everyday events	NS2.5 (p 72) Describes and compares chance events in social and experimental contexts	NS3.5 (p 73) Orders the likelihood of simple events on a number line from zero to one

Number Outcomes for Stage 4 to Stage 5

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Operations with Whole Numbers NS4.1 (p 56) Recognises the properties of special groups of whole numbers and applies a range of strategies to aid computation			
Integers NS4.2 (p 58) Compares, orders and calculates with integers			
Fractions, Decimals and Percentages NS4.3 (p 63) Operates with fractions, decimals, percentages, ratios and rates	**Rational Numbers** NS5.1.1 (p 65) Applies index laws to simplify and evaluate arithmetic expressions and uses scientific notation to write large and small numbers	**Rational Numbers** NS5.2.1 (p 67) Rounds decimals to a specified number of significant figures, expresses recurring decimals in fraction form and converts rates from one set of units to another	**§Real Numbers** §NS5.3.1 (p 68) Performs operations with surds and indices
	Consumer Arithmetic NS5.1.2 (p 70) Solves consumer arithmetic problems involving earning and spending money	**Consumer Arithmetic** NS5.2.2 (p 71) Solves consumer arithmetic problems involving compound interest, depreciation and successive discounts	
Probability NS4.4 (p 74) Solves probability problems involving simple events	**Probability** NS5.1.3 (p 75) Determines relative frequencies and theoretical probabilities		**Probability** NS5.3.2 (p 76) Solves probability problems involving compound events

§ - *recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course*

Patterns and Algebra Outcomes for Early Stage 1 to Stage 3

Substrand	Early Stage 1	Stage 1	Stage 2	Stage 3
Patterns and Algebra Students develop skills in creating, describing and recording number patterns as well as an understanding of the relationships between numbers	PAES1.1 Recognises, describes, creates and continues repeating patterns and number patterns that increase or decrease	PAS1.1 Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships	PAS2.1 (p 79) Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values	PAS3.1a (p 80) Records, analyses and describes geometric and number patterns that involve one operation using tables and words PAS3.1b (p 81) Constructs, verifies and completes number sentences involving the four operations with a variety of numbers

Patterns and Algebra Outcomes for Stage 4 to Stage 5

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3	
Algebraic Techniques PAS4.1 (p 82) Uses letters to represent numbers and translates between words and algebraic symbols **Number Patterns** PAS4.2 (p 83) Creates, records, analyses and generalises number patterns using words and algebraic symbols in a variety of ways				
Algebraic Techniques PAS4.3 (p 85) Uses the algebraic symbol system to simplify, expand and factorise simple algebraic expressions PAS4.4 (p 86) Uses algebraic techniques to solve linear equations and simple inequalities	**Algebraic Techniques** PAS5.1.1 (p 87) Applies the index laws to simplify algebraic expressions	**Algebraic Techniques** PAS5.2.1 (p 88) Simplifies, expands and factorises algebraic expressions involving fractions and negative and fractional indices PAS5.2.2 (p 90) Solves linear and simple quadratic equations, solves linear inequalities and solves simultaneous equations using graphical and analytical methods	**§Algebraic Techniques** §PAS5.3.1 (p 92) Uses algebraic techniques to simplify expressions, expand binomial products and factorise quadratic expressions §PAS5.3.2 (p 94) Solves linear, quadratic and simultaneous equations, solves and graphs inequalities, and rearranges literal equations	**#Curve Sketching and Polynomials** # PAS5.3.6 (p 107) Uses a variety of techniques to sketch a range of curves and describes the features of curves from the equation
Linear Relationships PAS4.5 (p 96) Graphs and interprets linear relationships on the number plane	**Coordinate Geometry** PAS5.1.2 (p 97) Determines the midpoint, length and gradient of an interval joining two points on the number plane and graphs linear and simple non-linear relationships from equations	**Coordinate Geometry** PAS5.2.3 (p 99) Uses formulae to find midpoint, distance and gradient and applies the gradient/intercept form to interpret and graph straight lines PAS5.2.4 (p 101) Draws and interprets graphs including simple parabolas and hyperbolas	**§Coordinate Geometry** §PAS5.3.3 (p 102) Uses various standard forms of the equation of a straight line and graphs regions on the number plane §PAS5.3.4 (p 103) Draws and interprets a variety of graphs including parabolas, cubics, exponentials and circles and applies coordinate geometry techniques to solve problems	# PAS5.3.7 (p 108) Recognises, describes and sketches polynomials, and applies the factor and remainder theorems to solve problems **#Functions and Logarithms** # PAS5.3.8 (p 109) Describes, interprets and sketches functions and uses the definition of a logarithm to establish and apply the laws of logarithms
		Graphs of Physical Phenomena PAS5.2.5 (p 105) Draws and interprets graphs of physical phenomena	**Graphs of Physical Phenomena** PAS5.3.5 (p 106) Analyses and describes graphs of physical phenomena	

 # - optional topics

§ - recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course

Data Outcomes for Early Stage 1 to Stage 3

Substrand	Early Stage 1	Stage 1	Stage 2	Stage 3
Data Students inform their inquiries through gathering, organising, tabulating and graphing data	DES1.1 Represents and interprets data displays made from objects and pictures	DS1.1 Gathers and organises data, displays data using column and picture graphs, and interprets the results	DS2.1 (p 112) Gathers and organises data, displays data using tables and graphs, and interprets the results	DS3.1 (p 113) Displays and interprets data in graphs with scales of many-to-one correspondence

Data Outcomes for Stage 4 to Stage 5

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Data Representation DS4.1 (p 114) Constructs, reads and interprets graphs, tables, charts and statistical information	**Data Representation and Analysis** DS5.1.1 (p 116) Groups data to aid analysis and constructs frequency and cumulative frequency tables and graphs		
Data Analysis and Evaluation DS4.2 (p 115) Collects statistical data using either a census or a sample, and analyses data using measures of location and range		**Data Analysis and Evaluation** DS5.2.1 (p 117) Uses the interquartile range and standard deviation to analyse data	

Measurement Outcomes for Early Stage 1 to Stage 3

Substrand	Early Stage 1	Stage 1	Stage 2	Stage 3
Length Students distinguish the attribute of length and use informal and metric units for measurement	MES1.1 Describes length and distance using everyday language and compares lengths using direct comparison	MS1.1 Estimates, measures, compares and records lengths and distances using informal units, metres and centimetres	MS2.1 (p 120) Estimates, measures, compares and records lengths, distances and perimeters in metres, centimetres and millimetres	MS3.1 (p 121) Selects and uses the appropriate unit and device to measure lengths, distances and perimeters
Area Students distinguish the attribute of area and use informal and metric units for measurement	MES1.2 Describes area using everyday language and compares areas using direct comparison	MS1.2 Estimates, measures, compares and records areas using informal units	MS2.2 (p 122) Estimates, measures, compares and records the areas of surfaces in square centimetres and square metres	MS3.2 (p 123) Selects and uses the appropriate unit to calculate area, including the area of squares, rectangles and triangles
Volume and Capacity Students recognise the attribute of volume and use informal and metric units for measuring capacity or volume	MES1.3 Compares the capacities of containers and the volumes of objects or substances using direct comparison	MS1.3 Estimates, measures, compares and records volumes and capacities using informal units	MS2.3 (p 128) Estimates, measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres	MS3.3 (p 130) Selects and uses the appropriate unit to estimate and measure volume and capacity, including the volume of rectangular prisms
Mass Students recognise the attribute of mass through indirect and direct comparisons, and use informal and metric units for its measurement	MES1.4 Compares the masses of two objects and describes mass using everyday language	MS1.4 Estimates, measures, compares and records the masses of two or more objects using informal units	MS2.4 (p 134) Estimates, measures, compares and records masses using kilograms and grams	MS3.4 (p 135) Selects and uses the appropriate unit and measuring device to find the mass of objects
Time Students develop an understanding of the passage of time, its measurement and representations, through the use of everyday language and experiences	MES1.5 Sequences events and uses everyday language to describe the duration of activities	MS1.5 Compares the duration of events using informal methods and reads clocks on the half-hour	MS2.5 (p 136) Reads and records time in one-minute intervals and makes comparisons between time units	MS3.5 (p 137) Uses twenty-four hour time and am and pm notation in real-life situations and constructs timelines

Measurement Outcomes for Stage 4 to Stage 5

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Perimeter and Area MS4.1 (p 124) Uses formulae and Pythagoras' theorem in calculating perimeter and area of circles and figures composed of rectangles and triangles	**Perimeter and Area** MS5.1.1 (p 126) Uses formulae to calculate the area of quadrilaterals and finds areas and perimeters of simple composite figures	**Perimeter and Area** MS5.2.1 (p 127) Finds areas and perimeters of composite figures	
Surface Area and Volume MS4.2 (p 131) Calculates surface area of rectangular and triangular prisms and volume of right prisms and cylinders		**Surface Area and Volume** MS5.2.2 (p 132) Applies formulae to find the surface area of right cylinders and volume of right pyramids, cones and spheres, and calculates the surface area and volume of composite solids	**Surface Area and Volume** MS5.3.1 (p 133) Applies formulae to find the surface area of pyramids, right cones and spheres
	Trigonometry MS5.1.2 (p 139) Applies trigonometry to solve problems (diagrams given) including those involving angles of elevation and depression	**Trigonometry** MS5.2.3 (p 140) Applies trigonometry to solve problems including those involving bearings	**§Trigonometry** §MS5.3.2 (p 141) Applies trigonometric relationships, sine rule, cosine rule and area rule in problem solving
Time MS4.3 (p 138) Performs calculations of time that involve mixed units			

\# - *optional topics*

§ - *recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course*

Space and Geometry Outcomes for Early Stage 1 to Stage 3

Substrand	Early Stage 1	Stage 1	Stage 2	Stage 3
Three-dimensional Space Students develop verbal, visual and mental representations of three-dimensional objects, their parts and properties, and different orientations	SGES1.1 Manipulates, sorts and represents three-dimensional objects and describes them using everyday language	SGS1.1 Sorts, describes and represents three-dimensional objects including cones, cubes, cylinders, spheres and prisms, and recognises them in pictures and the environment	SGS2.1 (p 145) Makes, compares, describes and names three-dimensional objects including pyramids, and represents them in drawings	SGS3.1 (p 146) Identifies three-dimensional objects, including particular prisms and pyramids, on the basis of their properties, and visualises, sketches and constructs them given drawings of different views
Two-dimensional Space Students develop verbal, visual and mental representations of lines, angles and two-dimensional shapes, their parts and properties, and different orientations	SGES1.2 Manipulates, sorts and describes representations of two-dimensional shapes using everyday language	SGS1.2 Manipulates, sorts, represents, describes and explores various two-dimensional shapes	SGS2.2a (p 149) Manipulates, compares, sketches and names two-dimensional shapes and describes their features SGS2.2b (p 150) Identifies, compares and describes angles in practical situations	SGS3.2a (p 151) Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties SGS3.2b (p 152) Measures, constructs and classifies angles
Position Students develop their representation of position through precise language and the use of grids and compass directions	SGES1.3 Uses everyday language to describe position and give and follow simple directions	SGS1.3 Represents the position of objects using models and drawings and describes using everyday language	SGS2.3 (p 165) Uses simple maps and grids to represent position and follow routes	SGS3.3 (p 166) Uses a variety of mapping skills

Space and Geometry Outcomes for Stage 4 to Stage 5

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Properties of Solids SGS4.1 (p 147) Describes and sketches three-dimensional solids including polyhedra, and classifies them in terms of their properties			
Angles SGS4.2 (p 153) Identifies and names angles formed by the intersection of straight lines, including those related to transversals on sets of parallel lines, and makes use of the relationships between them			**§Deductive Geometry** §SGS5.3.1 (p 159) Constructs arguments to prove geometrical results
Properties of Geometrical Figures SGS4.3 (p 154) Classifies, constructs, and determines the properties of triangles and quadrilaterals SGS4.4 (p 156) Identifies congruent and similar two-dimensional figures stating the relevant conditions		**Properties of Geometrical Figures** SGS5.2.1 (p 157) Develops and applies results related to the angle sum of interior and exterior angles for any convex polygon SGS5.2.2 (p 158) Develops and applies results for proving that triangles are congruent or similar	§SGS5.3.2 (p 160) Determines properties of triangles and quadrilaterals using deductive reasoning §SGS5.3.3 (p 162) Constructs geometrical arguments using similarity tests for triangles
			#Circle Geometry # SGS5.3.4 (p 163) Applies deductive reasoning to prove circle theorems and to solve problems

\# - *optional topics*

§ - *recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course*

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8 Continuum of Learning in Mathematics Years 7–10

8.1 K–10 Mathematics Scope and Continuum

The K–10 Mathematics Scope and Continuum (pp 30-39) is an overview of **key ideas** in each of the strands: Number, Patterns and Algebra, Data, Measurement, and Space and Geometry. For Early Stage 1 to Stage 3, the Scope and Continuum is organised into strands and substrands. For Stages 4 and 5, the Scope and Continuum is organised into strands and topics. These key ideas are also included on every page of the essential content that follows the Scope and Continuum.

The concepts in each of these strands are developed across the Stages to show how understanding in the early years needs to precede understanding in later years. In this way, the Scope and Continuum provides an overview of the sequence of learning for particular concepts in mathematics and links content typically taught in primary mathematics classrooms with content that is typically taught in secondary mathematics classrooms. It illustrates assumptions about prior learning and indicates pathways for further learning.

In this syllabus, it is generally the case that content is not repeated. This was intentional and is not meant to suggest that review and consolidation are no longer necessary. When programming, it will be necessary for teachers to determine the level of achievement of outcomes in previous Stages before planning new teaching and learning experiences. Students may be at different Stages for different strands of the Scope and Continuum. For example, a student may be working on Stage 4 content in Number but be working on Stage 3 content in the Space and Geometry strand.

It is not intended that the Scope and Continuum be used as a checklist of teaching ideas. Rather, a variety of learning experiences needs to be planned and presented to students to maximise opportunities for achievement of outcomes. Students need appropriate time to explore, experiment and engage with the underpinning concepts and principles of what they are to learn.

It should be noted that the Working Mathematically strand does not appear in this Scope and Continuum as it does not have content and key ideas. It is written as outcomes that are presented on pages 16 and 17.

Scope and Continuum of Key Ideas: Number

	Early Stage 1	Stage 1	Stage 2	Stage 3
Whole Numbers	Count forwards to 30, from a given number Count backwards from a given number, in the range 0 to 20	Count forwards and backwards by ones, twos and fives Count forwards and backwards by tens, on and off the decade	Count forwards and backwards by tens or hundreds, on and off the decade	Identify differences between Roman and Hindu-Arabic counting systems
	Compare, order, read and represent numbers to at least 20	Read, order and represent two- and three-digit numbers	Use place value to read, represent and order numbers up to four digits	Read, write and order numbers of any size using place value Record numbers in expanded notation
	Read and use the ordinal names to at least 'tenth'	Read and use the ordinal names to at least 'thirty-first'		Recognise the location of negative numbers in relation to zero
	Use the language of money	Sort, order and count money using face value	*Money concepts are developed further in Fractions and Decimals*	*Money concepts are developed further in Fractions and Decimals*
Addition and Subtraction	Combine groups to model addition Take part of a group away to model subtraction Compare groups to determine 'how many more'	Model addition and subtraction using concrete materials Develop a range of mental strategies and informal recording methods for addition and subtraction	Use a range of mental strategies for addition and subtraction involving two-, three- and four-digit numbers Explain and record methods for adding and subtracting	Select and apply appropriate mental, written or calculator strategies for addition and subtraction with counting numbers of any size
	Record addition and subtraction informally	Record number sentences using drawings, numerals, symbols and words	Use a formal written algorithm for addition and subtraction	
Multiplication and Division	Model equal groups or rows Group and share collections of objects equally	Rhythmic and skip count by ones, twos, fives and tens Model and use strategies for multiplication including arrays, equal groups and repeated addition Model and use strategies for division including sharing, arrays and repeated subtraction	Develop mental facility for number facts up to 10×10 Find multiples and squares of numbers Interpret remainders in division problems Determine factors for a given number	Select and apply appropriate mental, written or calculator strategies for multiplication and division Explore prime and composite numbers
	Record grouping and sharing informally	Record using drawings, numerals, symbols and words	Use mental and informal written strategies for multiplying or dividing a two-digit number by a one-digit operator	Use formal written algorithms for multiplication (limit operators to two-digit numbers) and division (limit operators to single digits)
Fractions and Decimals	Divide an object into two equal parts Recognise and describe halves	Model and describe a half or a quarter of a whole object Model and describe a half or a quarter of a collection of objects Use fraction notation $\frac{1}{2}$ and $\frac{1}{4}$	Model, compare and represent fractions with denominators 2, 4, and 8, followed by fractions with denominators 5, 10, and 100 Find equivalence between halves, quarters and eighths; fifths and tenths; tenths and hundredths	Model, compare and represent commonly used fractions (those with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100) Find equivalence between thirds, sixths and twelfths Express a mixed numeral as an improper fraction, and vice versa Add and subtract simple fractions where one denominator is a multiple of the other Multiply simple fractions by whole numbers. Calculate unit fractions of a number
			Model, compare and represent decimals to 2 decimal places Add and subtract decimals with the same number of decimal places (to 2 decimal places) Recognise percentages in everyday situations. Relate a common percentage to a fraction or decimal	Multiply and divide decimals by whole numbers in everyday contexts. Add and subtract decimals to three decimal places Calculate simple percentages of quantities
	Early money concepts are developed in Whole Numbers	*Money concepts are developed in Whole Numbers*	Perform calculations with money	Apply the four operations to money in real-life situations
Chance		Recognise the element of chance in familiar daily activities Use familiar language to describe the element of chance	Explore all possible outcomes in a simple chance situation Conduct simple chance experiments Collect data and compare likelihood of events in different contexts	Assign numerical values to the likelihood of simple events occurring Order the likelihood of simple events on a number line from 0 to 1

Scope and Continuum of Key Ideas: Number

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Operations with Whole Numbers Explore other counting systems Investigate groups of positive whole numbers Apply mental strategies to aid computation			
Integers Perform operations with directed numbers Simplify expressions involving grouping symbols and apply order of operations			
Operations with Whole Numbers Find squares/related square roots; cubes/related cube roots Use index notation for positive integral indices Determine and apply tests of divisibility Express a number as a product of its prime factors Divide two- or three-digit numbers by a two-digit number	**Rational Numbers** Define and use zero index and negative integral indices Develop the index laws arithmetically Use index notation for square and cube roots Express numbers in scientific notation (positive and negative powers of 10)		**§ Real Numbers** Use integers and fractions for index notation
Fractions, Decimals and Percentages Perform operations with fractions, decimals and mixed numerals Use ratios and rates to solve problems		**Rational Numbers** Express recurring decimals as fractions Round numbers to a specified number of significant figures Convert rates from one set of units to another	**§ Real Numbers** Define the system of real numbers distinguishing between rational and irrational numbers Perform operations with surds Convert between surd and index form
	Consumer Arithmetic Solve simple consumer problems including those involving earning and spending money Calculate simple interest and find compound interest using a calculator and table of values	**Consumer Arithmetic** Use compound interest formula Solve consumer arithmetic problems involving compound interest, depreciation, successive discounts	
Probability Determine the probability of simple events Solve simple probability problems Recognise complementary events	**Probability** Determine relative frequencies to estimate probabilities Determine theoretical probabilities		**Probability** Solve probability problems including two-stage and compound events § - *recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course*

Scope and Continuum of Key Ideas: Patterns and Algebra

	Early Stage 1	Stage 1	Stage 2	Stage 3
Patterns and Algebra	Recognise, describe, create and continue repeating patterns	Create, represent and continue a variety of number patterns and supply missing elements	Generate, describe and record number patterns using a variety of strategies	Build simple geometric patterns involving multiples Complete a table of values for geometric and number patterns Describe a pattern in words in more than one way
	Continue simple number patterns that increase or decrease	Build number relationships by relating addition and subtraction facts to at least 20 Make generalisations about number relationships	Build number relationships by relating multiplication and division facts to at least 10×10	
	Use the term 'is the same as' to describe equality of groups	Use the equals sign to record equivalent number relationships	Complete simple number sentences by calculating the value of a missing number	Construct, verify and complete number sentences involving the four operations with a variety of numbers

Scope and Continuum of Key Ideas: Patterns and Algebra

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Algebraic Techniques Use letters to represent numbers Translate between words and algebraic symbols and between algebraic symbols and words Recognise and use simple equivalent algebraic expressions **Number Patterns** Create, record and describe number patterns using words Use algebraic symbols to translate descriptions of number patterns Represent number pattern relationships as points on a grid			
Algebraic Techniques Use the algebraic symbol system to simplify, expand and factorise simple algebraic expressions Substitute into algebraic expressions	**Algebraic Techniques** Apply the index laws to simplify algebraic expressions (positive integral indices only)	**Algebraic Techniques** Simplify, expand and factorise algebraic expressions including those involving fractions or with negative and/or fractional indices	**§ Algebraic Techniques** Use algebraic techniques to simplify expressions, expand binomial products and factorise quadratic expressions
Solve linear equations and word problems using algebra		Solve linear and simple quadratic equations of the form $ax^2 = c$	Solve quadratic equations by factorising, completing the square, or using the quadratic formula
Solve simple inequalities		Solve linear inequalities	Solve a range of inequalities and rearrange literal equations
		Solve simultaneous equations using graphical and analytical methods for simple examples	Solve simultaneous equations including quadratic equations
Linear Relationships Interpret the number plane and locate ordered pairs	**Coordinate Geometry** Use a diagram to determine midpoint, length and gradient of an interval joining two points on the number plane	**Coordinate Geometry** Use midpoint, distance and gradient formulae	
Graph and interpret linear relationships created from simple number patterns and equations	Graph linear and simple non-linear relationships from equations	Apply the gradient/intercept form to interpret and graph straight lines	**§ Coordinate Geometry** Use and apply various standard forms of the equation of a straight line, and graph regions on the number plane
		Draw and interpret graphs including simple parabolas and hyperbolas	Draw and interpret a variety of graphs including parabolas, cubics, exponentials and circles Solve coordinate geometry problems
		Graphs of Physical Phenomena Draw and interpret graphs of physical phenomena	**Graphs of Physical Phenomena** Analyse and describe graphs of physical phenomena
			#Curve Sketching and Polynomials Sketch a range of polynomials Add, subtract, multiply and divide polynomials Apply the factor and remainder theorems
			# Functions and Logarithms Define functions Use function notation Determine inverse functions Establish and apply the laws of logarithms

\# *- optional topics*

§ *- recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course*

Scope and Continuum of Key Ideas: Data

	Early Stage 1	Stage 1	Stage 2	Stage 3
Data	Collect data about students and their environment Organise actual objects or pictures of the objects into a data display	Gather and record data using tally marks Display the data using concrete materials and pictorial representations Use objects or pictures as symbols to represent other objects, using one-to-one correspondence	Conduct surveys, classify and organise data using tables Construct vertical and horizontal column graphs and picture graphs	Draw picture, column, line and divided bar graphs using scales of many-to-one correspondence
	Interpret data displays made from objects and pictures	Interpret information presented in picture graphs and column graphs	Interpret data presented in tables, column graphs and picture graphs	Read and interpret sector (pie) graphs Read and interpret graphs with scales of many-to-one correspondence
				Determine the mean (average) for a small set of data

Scope and Continuum of Key Ideas: Data

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Data Representation			
Draw, read and interpret graphs (line, sector, travel, step, conversion, divided bar, dot plots and stem-and-leaf plots), tables and charts			
Distinguish between types of variables used in graphs			
Identify misrepresentation of data in graphs			
	Data Representation and Analysis		
Construct frequency tables	Construct frequency tables for grouped data		
Draw frequency histograms and polygons			
Data Analysis and Evaluation			
Use sampling and census			
Make predictions from samples and diagrams		**Data Analysis and Evaluation**	
Analyse data using mean, mode, median and range	Find mean and modal class for grouped data	Determine the upper and lower quartiles of a set of scores	
	Determine cumulative frequency	Construct and interpret box-and-whisker plots	
	Find median using a cumulative frequency table or polygon	Find the standard deviation of a set of scores using a calculator	
		Use the terms 'skew' and 'symmetrical' to describe the shape of a distribution	

Scope and Continuum of Key Ideas: Measurement

	Early Stage 1	Stage 1	Stage 2	Stage 3
Length	Identify and describe the attribute of length Compare lengths directly by placing objects side-by-side and aligning the ends	Use informal units to estimate and measure length and distance by placing informal units end-to-end without gaps or overlaps Recognise the need for metres and centimetres, and use them to estimate and measure length and distance	Estimate, measure, compare and record lengths and distances using metres, centimetres and/or millimetres Convert between metres and centimetres, and centimetres and millimetres	Select and use the appropriate unit and device to measure lengths, distances and perimeters Convert between metres and kilometres; millimetres, centimetres and metres
			Estimate and measure the perimeter of two-dimensional shapes	Calculate and compare perimeters of squares, rectangles and equilateral and isosceles triangles
	Record comparisons informally	Record measurements by referring to the number and type of informal or formal units used	Record lengths and distances using decimal notation to two places	Record lengths and distances using decimal notation to three places
Area	Identify and describe the attribute of area	Use appropriate informal units to estimate and measure area	Recognise the need for square centimetres and square metres to measure area	Select and use the appropriate unit to calculate area Recognise the need for square kilometres and hectares
	Estimate the larger of two areas and compare using direct comparison	Compare and order two or more areas	Estimate, measure, compare and record areas in square centimetres and square metres	Develop formulae in words for finding area of squares, rectangles and triangles
	Record comparisons informally	Record measurements by referring to the number and type of informal units used		
Volume and Capacity	Identify and describe the attributes of volume and capacity	Use appropriate informal units to estimate and measure volume and capacity	Recognise the need for a formal unit to measure volume and capacity	Select the appropriate unit to measure volume and capacity Recognise the need for cubic metres
	Compare the capacities of two containers using direct comparison Compare the volumes of two objects by direct observation	Compare and order the capacities of two or more containers and the volumes of two or more models or objects	Estimate, measure, compare and record volumes and capacities using litres and millilitres Measure the volume of models in cubic centimetres	Estimate and measure the volume of rectangular prisms
			Convert between litres and millilitres	Determine the relationship between cubic centimetres and millilitres
	Record comparisons informally	Record measurements by referring to the number and type of informal units used		Record volume and capacity using decimal notation to three decimal places
Mass	Identify and describe the attribute of mass	Estimate and measure the mass of an object using an equal arm balance and appropriate informal units	Recognise the need for a formal unit to measure mass	Select and use the appropriate unit and device to measure mass Recognise the need for tonnes
	Compare the masses of two objects by pushing, pulling or hefting or using an equal arm balance	Compare and order two or more objects according to mass	Estimate, measure, compare and record masses using kilograms and grams	
				Convert between kilograms and grams and between kilograms and tonnes
	Record comparisons informally	Record measurements by referring to the number and type of informal units used		Record mass using decimal notation to three decimal places
Time	Describe the duration of events using everyday language Sequence events in time Name days of the week and seasons	Use informal units to measure and compare the duration of events Name and order the months and seasons of the year Identify the day and date on a calendar	Recognise the coordinated movements of the hands on a clock Read and record time using digital and analog notation Convert between units of time	Convert between am/pm notation and 24-hour time Compare various time zones in Australia, including during daylight saving Draw and interpret a timeline using a scale
	Tell time on the hour on digital and analog clocks	Tell time on the hour and half-hour on digital and analog clocks	Read and interpret simple timetables, timelines and calendars	Use timetables involving 24-hour time

Scope and Continuum of Key Ideas: Measurement

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Perimeter and Area Describe the limits of accuracy of measuring instruments			
Convert between metric units of length			
Develop formulae and use to find the area and perimeter of triangles, rectangles and parallelograms	**Perimeter and Area** Develop formulae and use to find the area of rhombuses, trapeziums and kites	**Perimeter and Area** Find area and perimeter of more complex composite figures	
Find the areas of simple composite figures Investigate and find the area and circumference of circles Convert between metric units of area	Find the area and perimeter of simple composite figures consisting of two shapes including quadrants and semicircles		
Surface Area and Volume Find the surface area of rectangular and triangular prisms		**Surface Area and Volume** Find surface area of cylinders and composite solids	**Surface Area and Volume** Apply formulae for the surface area of pyramids, right cones and spheres Explore and use similarity relationships for area and volume
Find the volume of right prisms and cylinders		Find the volume of pyramids, cones, spheres and composite solids	
Convert between metric units of volume			
Perimeter and Area Apply Pythagoras' theorem	**Trigonometry** Use trigonometry to find sides and angles in right-angled triangles	**Trigonometry** Solve further trigonometry problems including those involving three-figure bearings	**§ Trigonometry** Determine the exact trigonometric ratios for 30º, 45º, 60º
	Solve problems involving angles of elevation and angles of depression from diagrams		Apply relationships in trigonometry for complementary angles and tan in terms of sin and cos
			Determine trigonometric ratios for obtuse angles
			Sketch sine and cosine curves
			Explore trigonometry with non-right-angled triangles: sine rule, cosine rule and area rule
			Solve problems involving more than one triangle using trigonometry
Time Perform operations involving time units			
Use international time zones to compare times			
Interpret a variety of tables and charts related to time			

§ - *recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course*

Scope and Continuum of Key Ideas: Space and Geometry

	Early Stage 1	Stage 1	Stage 2	Stage 3
Three-dimensional Space	Manipulate and sort three-dimensional objects found in the environment Describe features of three-dimensional objects using everyday language Use informal names for three-dimensional objects	Name, describe, sort and model cones, cubes, cylinders, spheres and prisms Recognise three-dimensional objects in pictures and the environment, and presented in different orientations Recognise that three-dimensional objects look different from different views	Name, describe, sort, make and sketch prisms, pyramids, cylinders, cones and spheres Create nets from everyday packages Describe cross-sections of three-dimensional objects	Identify three-dimensional objects, including particular prisms and pyramids, on the basis of their properties Construct three-dimensional models given drawings of different views
Two-dimensional Space	Manipulate, sort and describe two-dimensional shapes Identify and name circles, squares, triangles and rectangles in pictures and the environment, and presented in different orientations Represent two-dimensional shapes using a variety of materials	Identify, name, compare and represent hexagons, rhombuses and trapeziums presented in different orientations	Identify and name pentagons, octagons and parallelograms presented in different orientations Compare and describe special groups of quadrilaterals	Identify right-angled, isosceles, equilateral and scalene triangles Identify and draw regular and irregular two-dimensional shapes Identify and name parts of a circle
		Make tessellating designs using flips, slides and turns	Make tessellating designs by reflecting, translating and rotating	Enlarge and reduce shapes, pictures and maps
		Identify a line of symmetry	Find all lines of symmetry for a two-dimensional shape	Identify shapes that have rotational symmetry
	Identify and draw straight and curved lines	Identify and name parallel, vertical and horizontal lines		
		Identify corners as angles	Recognise openings, slopes and turns as angles Describe angles using everyday language and the term 'right'	Classify angles as right, acute, obtuse, reflex, straight or a revolution
		Compare angles by placing one angle on top of another	Compare angles using informal means	Measure in degrees and construct angles using a protractor
Position	Give and follow simple directions	Represent the position of objects using models and drawings	Use simple maps and grids to represent position and follow routes	Interpret scales on maps and plans Make simple calculations using scale
	Use everyday language to describe position	Describe the position of objects using everyday language, including 'left' and 'right'	Determine the directions N, S, E and W, NE, NW, SE and SW, given one of the directions Describe the location of an object on a simple map using coordinates or directions	

Scope and Continuum of Key Ideas: Space and Geometry

Stage 4	Stage 5.1	Stage 5.2	Stage 5.3
Properties of Solids Determine properties of three-dimensional objects Investigate Platonic solids Investigate Euler's relationship for convex polyhedra Make isometric drawings			
Properties of Geometrical Figures Classify, construct and determine properties of triangles and quadrilaterals		**Properties of Geometrical Figures** Verify the properties of special quadrilaterals using congruent triangles	**§ Deductive Geometry** Use deductive geometry to prove properties of special triangles and quadrilaterals
Investigate similar figures and interpret and construct scale drawings Identify congruent figures		Identify similar triangles and describe their properties Apply tests for congruent triangles	Construct geometrical arguments using similarity tests for triangles
Complete simple numerical exercises based on geometrical properties		Use simple deductive reasoning in numerical and non-numerical problems	Construct proofs of geometrical relationships involving congruent or similar triangles
Angles Classify angles and determine angle relationships Construct parallel and perpendicular lines and determine associated angle properties		Establish sum of exterior angles result and sum of interior angles result for polygons	
			# Circle Geometry Deduce chord, angle, tangent and secant properties of circles

 # - optional topics

§ - recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course

8.2 Stage Statements for Stages 2 to 5

Stage statements describe what students typically know and can do as a consequence of having undertaken the syllabus content prescribed for the Stage.

Stage 2

Students who have achieved Stage 2 outcomes demonstrate their problem-solving skills using a range of strategies to deal with simple spatial, measurement and numerical problems. They pose questions that can be explored using mathematics. They develop and check solutions using appropriate mental or written strategies or technology. They use some mathematical terminology to describe or represent mathematical ideas and link their learning to other experiences.

Students use place value to count, order, read and record numbers up to four digits. They use estimation and employ mental strategies to solve problems. They demonstrate mental facility with multiples of numbers up to 10×10 and use informal written strategies for multiplication and division. They solve addition and subtraction problems using mental and written strategies, including the formal written algorithm. They model, compare and represent commonly used fractions and related decimals and recognise percentages in everyday situations. They perform simple calculations with money and use estimation to check their solutions. They order events from least likely to most likely and identify and record all possible outcomes for a simple chance experiment. They generate, describe and record number patterns using a variety of strategies and complete simple number sentences by calculating missing values.

Students conduct surveys, and classify and organise data to answer a specific question they have posed. They present the information in tables and graphs and interpret the results.

Students recognise the need for formal units to measure perimeter, area, volume, capacity and mass. They use particular formal units to estimate and measure to the nearest unit. They read and record time in one-minute intervals, make comparisons between time units, and interpret calendars, simple timetables and timelines.

Students identify, manipulate and compare groups of three-dimensional objects and two-dimensional shapes and describe their features using appropriate mathematical terminology. They make and describe tessellating designs, identify perpendicular lines and find line/s of symmetry for a given shape. They are aware of angles in the environment and measure them using informal means. Students use coordinates to describe position and give and follow directions using compass points.

Stage 3

Students who have achieved Stage 3 outcomes extend mathematical investigations using appropriate problem-solving strategies, including the selection and use of appropriate technology. They use appropriate mathematical terminology and some conventions when representing mathematical situations and give a valid reason for supporting one possible solution to a problem over another. They are able to apply a familiar solution method to new problems. Students appreciate that mathematics involves observing, representing and generalising patterns and relationships.

Students read, write, represent and order numbers of any size using place value. They select and apply appropriate strategies for the four operations and interpret their solutions in the context of a problem. Students compare, order and perform calculations involving simple fractions, decimals and simple percentages. They assign numerical values to the likelihood of simple events occurring and order them on the number line. Students record using tables, and analyse and describe geometric and number patterns that involve one operation. Students construct, verify and complete number sentences using the four operations with a variety of numbers.

Students gather, organise, display, read and interpret data and make judgements in relation to this data. They read and interpret picture, simple line, pie and divided bar graphs with scales. They utilise data to find the average score.

Students select and use the appropriate device and unit for measuring. They convert measurements from one unit to another and record in decimal notation. They estimate and measure volume and capacity, including the volume of rectangular prisms, in cubic centimetres and cubic metres. Students use 24-hour time, am and pm notation and construct timelines and simple timetables. They use Australian time zones to solve simple problems related to time differences.

Students construct and classify three-dimensional objects and two-dimensional shapes and compare and describe their properties using strategies such as recognising symmetry and measuring angles and dimensions. They make simple calculations using scale and use a variety of mapping skills.

Stage 4

Students who have achieved Stage 4 outcomes use mathematical terminology, algebraic notation, diagrams, text and tables to communicate mathematical ideas, and link concepts and processes within and between mathematical contexts. They apply their mathematical skills and understanding in analysing real-life situations and in systematically formulating questions or problems that they then explore and solve, using technology where appropriate. In solving particular problems, they compare the strengths and weaknesses of different strategies and solutions.

Students have developed a range of mental strategies to enhance their computational skills. They operate competently with directed numbers, fractions, percentages, mixed numerals and decimals and apply these in a range of practical contexts, including problems related to discounts and profit and loss. They are familiar with the concepts of ratio, rates and the probability of simple and complementary events and apply these when solving problems. They use index notation for numbers with positive integral indices and explore prime factorisation, squares and related square roots, and cubes and related cube roots. Students investigate special groups of positive whole numbers, divisibility tests and other counting systems.

Extending and generalising number patterns leads students into an understanding of the use of pronumerals and the language of algebra, including the use of index notation. Students simplify algebraic expressions, substitute into algebraic expressions and formulae, and expand and factorise algebraic expressions. They solve simple linear equations, inequalities, and word problems. They develop tables of values from simple relationships and illustrate these relationships on the number plane.

Students construct and interpret line, sector, travel, step and conversion graphs, dot plots, stem-and-leaf plots, divided bar graphs, and frequency tables and histograms. In analysing data, they consider both discrete and continuous variables, sampling versus census, prediction and possible misrepresentation of data, and calculate the mean, mode, median and range.

Students find the area and perimeter of a variety of polygons, circles, and simple composite figures, the surface area and volume of rectangular and triangular prisms, and the volume of cylinders and right prisms. Pythagoras' theorem is used to calculate the distance between two points. They describe the limit of accuracy of their measures, interpret and use tables and charts related to time, and apply their understanding of Australian and world time zones to solve problems.

Their knowledge of the properties of two- and three-dimensional geometrical figures, angles, parallel lines, perpendicular lines, congruent figures, similar figures and scale drawings enables them to solve numerical exercises on finding unknown lengths and angles in figures.

Stage 5

5.1

Students who have achieved Stage 5.1 outcomes explain and verify mathematical relationships, ask and explore questions which can be solved using mathematics, and link mathematical ideas to existing knowledge and understanding. They use mathematical language and notation to explain mathematical ideas, and interpret tables, diagrams and text in mathematical situations.

Students apply their knowledge of percentages, fractions and decimals to problems involving consumer situations related to earning and spending money, and simple and compound interest. They simplify and evaluate arithmetic expressions using index laws and express numbers in scientific notation using both positive and negative powers of ten. Students determine relative frequency and theoretical probability.

Students apply the index laws to simplify algebraic expressions. They determine the midpoint, length and gradient of intervals on the number plane and draw graphs of linear and simple non-linear relationships.

Their statistical skills are extended to include grouping data into class intervals and constructing and interpreting cumulative frequency tables, histograms and polygons.

Skills in measurement are further developed to include the use of formulae when calculating the area and perimeter of composite figures. Students apply right-angled triangle trigonometry to practical situations including those involving angles of elevation and depression.

5.2

Students who have achieved the syllabus outcomes, up to and including Stage 5.2 outcomes, ask questions that can be explored using mathematics, and use mathematical arguments to reach and justify conclusions. When communicating mathematical ideas, they use appropriate mathematical language and algebraic, statistical and other notations and conventions in written, oral or graphical form. Students use suitable problem-solving strategies which include selecting and organising key information and they extend their inquiries by identifying and working on related problems.

Students apply their knowledge of percentages, fractions and decimals to problems involving conversion of rates and consumer situations related to compound interest, depreciation and successive discounts. They express recurring decimals as fractions, and round numbers to a specified number of significant figures.

Students solve non-routine problems in algebra and apply the index laws to simplify, expand and factorise algebraic expressions. They solve linear equations and simple quadratic equations, inequalities and simultaneous equations. On the number plane they draw and interpret graphs of straight lines, simple parabolas, hyperbolas and graphs of physical phenomena. Formulae are used to find distance, gradient and midpoint.

Statistical skills are extended to include descriptions of distributions and the construction of box-and-whisker plots. Student analysis of data includes determining upper and lower quartiles and standard deviation.

Students extend their skills in measurement to calculations of the area and perimeter of complex composite figures, the volume of pyramids, cones, spheres and composite solids, and the surface area of cylinders and composite solids. In geometry, they use deductive reasoning in numerical and non-numerical problems drawing on their knowledge of the properties of similar and congruent triangles, the angle properties of polygons and the properties of quadrilaterals, including diagonal properties.

5.3

Students who have achieved the syllabus outcomes, up to and including Stage 5.3 outcomes, use deductive reasoning in problem solving and in presenting arguments and formal proofs. They interpret and apply formal definitions and generalisations and connect and apply mathematical ideas within and across topics.

Students calculate the probability of compound events, operate with irrational numbers and extend their knowledge of the number system to include all real numbers. They apply algebra to analysing and describing physical phenomena and rates of change. Algebraic skills are extended to expanding binomial products, factorising quadratic expressions, and solving literal equations, inequalities, quadratic and simultaneous equations. They generate, describe and graph equations of straight lines, parabolas, cubics, hyperbolas, circles and exponential functions, and are able to graph regions determined by inequalities.

Students calculate the surface areas of pyramids, cones and spheres and explore and use similarity relationships for area and volume. They determine exact trigonometric ratios for 30º, 45º and 60º, extend trigonometric ratios to obtuse angles and sketch sine and cosine curves. Students apply the sine and cosine rules for finding unknown angles and/or sides in non-right-angled triangles.

Their knowledge of a wide range of geometrical facts and relationships is used to prove general statements in geometry, extending the concepts of similarity and congruence to a more generalised application. Students prove Pythagoras' theorem and the properties of triangles and quadrilaterals.

9 Content for Stages 2 to 5

Introduction

This section contains the content for Stages 2 to 5 so that teachers can meet the learning needs of students in Years 7 to 10. Within each strand and substrand or topic, the outcomes, key ideas, content, background information, and advice about language are presented in tables as follows. The content is comprised of the statements of knowledge and skills in the left hand column and the statements about Working Mathematically in the right hand column.

For Stages 2 and 3, there are some substrands that contain the development of several concepts. To enable ease of programming, the content has been separated into two units. The first unit typically contains early concept development and the second unit continues with further development of the concepts.

<table>
<tr><td colspan="2">Substrand Stage</td></tr>
<tr><td>Outcome Code

A statement of the outcome.</td><td>Key Ideas

A list of the key ideas to be addressed that summarise the content statements listed below in both the left and right columns. These are also listed on the Scope and Continuum.</td></tr>
<tr><td>Knowledge and Skills
Students learn about
A set of statements related to the knowledge and skills students need to understand and apply in order to achieve the outcome.
These are generally presented as a hierarchy of concept development; however, separate statements would typically be grouped and addressed together when planning teaching and learning experiences.</td><td>Working Mathematically
Students learn to
A sample set of statements that incorporate Working Mathematically processes into the knowledge and skills listed in the left hand column.
Teachers are encouraged to extend this list of statements by creating their own Working Mathematically experiences for students to engage with each of the five processes (Questioning, Applying Strategies, Communicating, Reasoning and Reflecting).</td></tr>
<tr><td colspan="2">Understanding is encompassed in the development of concepts and processes in both of these columns.</td></tr>
<tr><td colspan="2">Background Information

Listed here, where appropriate, is information about the mathematics involved, historical and cultural connections, and links with other key learning areas and other strands of mathematics.</td></tr>
<tr><td colspan="2">Language

Advice about language and literacy that may assist student engagement and understanding of the content in the unit.</td></tr>
</table>

9.1 Working Mathematically

Working Mathematically encompasses five interrelated processes. These processes come into play when developing new skills and concepts and also when applying existing knowledge to solve routine and non-routine problems both within and beyond mathematics. At times the focus may be on a particular process or group of processes, but often the five processes overlap. While this strand has a set of separate outcomes, it is integrated into the content of each of the five content strands in the syllabus.

Working Mathematically provides opportunities for students to engage in genuine mathematical activity and to develop the skills to become flexible and creative users of mathematics.

The five processes for Working Mathematically are:

Questioning	Students ask questions in relation to mathematical situations and their mathematical experiences. Encouraging students to ask questions builds on and stimulates their curiosity and interest in mathematics. 'I wonder if' and 'what if' types of questions encourage students to make conjectures and/or predictions.
Applying Strategies	Students develop, select and use a range of strategies, including the selection and use of appropriate technology, to explore and solve problems.
Communicating	Students develop and use appropriate language and representations to formulate and express mathematical ideas in written, oral and diagrammatic form.
Reasoning	Students develop and use processes for exploring relationships, checking solutions and giving reasons to support their conclusions. Students also need to develop and use logical reasoning, proof and justification.
Reflecting	Students reflect on their experiences and critical understanding to make connections with, and generalisations about, existing knowledge and understanding. Students make connections with the use of mathematics in the real world by identifying where, and how, particular ideas and concepts are used.

Examples of learning experiences for each of the processes for Working Mathematically are embedded in the right-hand column of the content for each outcome in the Number, Patterns and Algebra, Data, Measurement, and Space and Geometry strands.

9.2 Number

The skills developed in the Number strand are fundamental to all other strands of this Mathematics syllabus and are developed across the Stages from Early Stage 1 to Stage 5.3. Numbers, in their various forms, are used to quantify and describe the world. From Early Stage 1 there is an emphasis on the development of number sense and confidence and competence in using mental, written and calculator techniques for solving appropriate problems. Algorithms are introduced after students have gained a firm understanding of basic concepts including place value, and have developed mental strategies for computing with two- and three-digit numbers. Approximation is important and the systematic use of estimation is to be encouraged at all times. Students should always check that their answers 'make sense' in the context of the problems they are solving.

The use of mental computation strategies should be developed at all Stages. Calculators can be used to investigate number patterns and relationships and facilitate the solution of real problems with measurements and quantities not easy to handle with mental or written techniques.

Other areas of this strand are important for the development of a sound understanding of number and its applications. In Stages 2, 3, 4 and 5, students apply their number skills to a variety of consumer arithmetic and other practical problems, important life skills that relate to numeracy development. Ratio and rates underpin proportional reasoning needed for problem solving and the development of concepts and skills in other areas of mathematics, such as trigonometry, similarity and gradient. It should be noted that Chance concepts have been introduced in Stage 1. The language of chance and the important ideas related to the probability of various events are developed across all subsequent Stages.

The Number strand for Stages 2 and 3 is organised into five substrands:

- Whole Numbers
- Addition and Subtraction
- Multiplication and Division
- Fractions and Decimals
- Chance.

In Stage 4, the Number strand is organised into the topics: Operations with Whole Numbers, Integers, Fractions, Decimals and Percentages; and Probability. Stage 5.1 contains the topics Rational Numbers, Consumer Arithmetic, and Probability. Stage 5.2 contains the topics Rational Numbers and Consumer Arithmetic and Stage 5.3 the topics Real Numbers and Probability.

Summary of Number Outcomes for Stages 2 to 5 with page references

Whole Numbers

NS2.1 Counts, orders, reads and records numbers up to four digits (p 48)

NS3.1 Orders, reads and writes numbers of any size (p 49)

Addition and Subtraction

NS2.2 Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers (p 50)

NS3.2 Selects and applies appropriate strategies for addition and subtraction with counting numbers of any size (p 52)

Multiplication and Division

NS2.3 Uses mental and informal written strategies for multiplication and division (p 53)

NS3.3 Selects and applies appropriate strategies for multiplication and division (p 55)

Operations with Whole Numbers

NS4.1 Recognises the properties of special groups of whole numbers and applies a range of strategies to aid computation (p 56)

Integers

NS4.2 Compares, orders and calculates with integers (p 58)

Fractions and Decimals

NS2.4 Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages (p 59)

NS3.4 Compares, orders and calculates with decimals, simple fractions and simple percentages (p 61)

Fractions, Decimals and Percentages

NS4.3 Operates with fractions, decimals, percentages, ratios and rates (p 63)

Rational Numbers

NS5.1.1 Applies index laws to simplify and evaluate arithmetic expressions and uses scientific notation to write large and small numbers (p 65)

NS5.2.1 Rounds decimals to a specified number of significant figures, expresses recurring decimals in fraction form and converts rates from one set of units to another (p 67)

§Real Numbers

§NS5.3.1 Performs operations with surds and indices (p 68)

Consumer Arithmetic

NS5.1.2 Solves consumer arithmetic problems involving earning and spending money (p 70)

NS5.2.2 Solves consumer arithmetic problems involving compound interest, depreciation and successive discounts (p 71)

Chance

NS2.5 Describes and compares chance events in social and experimental contexts (p 72)

NS3.5 Orders the likelihood of simple events on a number line from zero to one (p 73)

Probability

NS4.4 Solves probability problems involving simple events (p 74)

NS5.1.3 Determines relative frequencies and theoretical probabilities (p 75)

NS5.3.2 Solves probability problems involving compound events (p 76)

(§ recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course)

Whole Numbers	**Stage 2**
NS2.1 Counts, orders, reads and records numbers up to four digits	**Key Ideas** Use place value to read, represent and order numbers up to four digits Count forwards and backwards by tens or hundreds, on and off the decade
Knowledge and Skills **Students learn about** • representing numbers up to four digits using numerals, words, objects and digital displays • identifying the number before and after a given two-, three- or four-digit number • applying an understanding of place value and the role of zero to read, write and order numbers up to four digits • stating the place value of digits in two-, three- or four-digit numbers eg 'in the number 3426, the 3 represents 3000 or 3 thousands' • ordering a set of four-digit numbers in ascending or descending order • using the symbols for 'is less than' (<) and 'is greater than' (>) to show the relationship between two numbers • counting forwards and backwards by tens or hundreds, on and off the decade eg 1220, 1230, 1240 (on the decade); 423, 323, 223 (off the decade) • recording numbers up to four digits using expanded notation eg 5429 = 5000+400+20+9 • rounding numbers to the nearest ten, hundred or thousand when estimating	**Working Mathematically** **Students learn to** • pose problems involving four-digit numbers *(Questioning)* • interpret four-digit numbers used in everyday contexts *(Communicating)* • compare and explain the relative size of four-digit numbers *(Applying Strategies, Communicating)* • make the largest and smallest number given any four digits *(Applying Strategies)* • solve a variety of problems using problem-solving strategies, including: - trial and error - drawing a diagram - working backwards - looking for patterns - using a table *(Applying Strategies, Communicating)*

Background Information

Students should be encouraged to develop different counting strategies eg if they are counting a large number of shells they can count out groups of ten and then count the groups.

The place value of digits in various numerals is investigated. Students should understand, for example, that the five in 35 represents five ones but the 5 in 53 represents five tens.

The convention for writing numbers of more than four digits requires that they have a space (and not a comma) to the left of each group of three digits, when counting from the Units column.

Language

The word 'and' is used between the hundreds and the tens when reading a number, but not between other places eg three thousand, six hundred and sixty-three.

The word 'round' has different meanings in different contexts eg the plate is round, round 23 to the nearest ten.

The word 'place' has different meanings in everyday language to those used in a mathematical context.

Whole Numbers	**Stage 3**
NS3.1 Orders, reads and writes numbers of any size	**Key Ideas** Read, write and order numbers of any size using place value Record numbers in expanded notation Recognise the location of negative numbers in relation to zero Identify differences between Roman and Hindu-Arabic counting systems

Knowledge and Skills

Students learn about

- applying an understanding of place value and the role of zero to read, write and order numbers of any size
- stating the place value of any digit in large numbers
- ordering numbers of any size in ascending or descending order
- recording large numbers using expanded notation eg 59 675 = 50 000+9000+600+70+5
- rounding numbers when estimating
- recognising different abbreviations of numbers used in everyday contexts eg $350K represents $350 000
- recognising the location of negative numbers in relation to zero and locating them on a number line
- recognising, reading and converting Roman numerals used in everyday contexts eg books, clocks, films
- identifying differences between the Roman and Hindu-Arabic systems of recording numbers

Working Mathematically

Students learn to

- ask questions that extend understanding of numbers eg 'What if …?' *(Questioning)*
- use large numbers in real-life situations eg population, money applications *(Reflecting, Applying Strategies)*
- interpret information from the Internet, media, environment and other sources that use large numbers *(Communicating)*
- investigate negative numbers and the number patterns created when counting backwards on a calculator *(Applying Strategies)*
- link negative numbers with subtraction *(Reflecting)*
- interpret negative whole numbers in everyday contexts eg temperature *(Communicating, Reflecting)*
- record numerical data in a simple spreadsheet *(Applying Strategies)*
- apply strategies to estimate large quantities *(Applying Strategies)*

Background Information

The convention for writing numbers of more than four digits requires that they have a space (and not a comma) to the left of each group of three digits, when counting from the Units column.

Students need to develop an understanding of place value relationships such as 10 thousand = 100 hundreds = 1000 tens = 10 000 ones.

The abbreviation K comes from the Greek word *khilioi* meaning thousand. It is used in many job advertisements (eg a salary of $70K) and as an abbreviation for the size of computer files eg 26K (kilobytes).

When identifying Roman numerals in everyday contexts it needs to be noted that the number four is sometimes represented using IIII instead of IV.

Addition and Subtraction **Stage 2**

NS2.2

Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

Key Ideas

Use a range of mental strategies for addition and subtraction involving two-, three- and four-digit numbers

Explain and record methods for adding and subtracting

Use a formal written algorithm for addition and subtraction

Knowledge and Skills

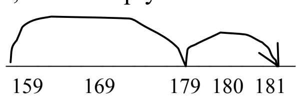
Students learn about

- using mental strategies for addition and subtraction involving two-, three- and four-digit numbers, including
  - the jump strategy
    eg 23+35; 23+30 = 53, 53+5 = 58
  - the split strategy
    eg 23+35; 20+30+3+5 is 58
  - the compensation strategy
    eg 63+29; 63+30 is 93, subtract 1, to obtain 92
  - using patterns to extend number facts
    eg 5–2 = 3, so 500–200 is 300
  - bridging the decades
    eg 34+17; 34+10 is 44, 44+7 = 51
  - changing the order of addends to form multiples of 10 eg 16+8+4; add 16 and 4 first
- recording mental strategies

  eg 159+22

  'I added 20 to 159 to get 179, then I added 2 more to get 181.'

  or, on an empty number line



- adding and subtracting two or more numbers, with and without trading, using concrete materials and recording their method
- using a formal written algorithm and applying place value to solve addition and subtraction problems, involving two-, three- and four-digit numbers

eg	134	2459	568	1353
	+ 253	+ 138	– 322	– 168

Working Mathematically

Students learn to

- pose problems that can be solved using addition and subtraction, including those involving money *(Questioning)*
- ask 'What is the best method to find a solution to this problem?' *(Questioning)*
- select and use mental, written or calculator methods to solve addition and subtraction problems *(Applying Strategies)*
- solve a variety of problems using problem-solving strategies, including:
  - trial and error
  - drawing a diagram
  - working backwards
  - looking for patterns
  - using a table *(Applying Strategies, Communicating)*
- use estimation to check solutions to addition and subtraction problems, including those involving money *(Reflecting, Applying Strategies)*
- check the reasonableness of a solution to a problem by relating it to an original estimation *(Reasoning)*
- check solutions using the inverse operation or a different method *(Applying Strategies, Reasoning)*
- explain how an answer was obtained for an addition or subtraction problem *(Communicating, Reasoning)*
- reflect on own method of solution for a problem, considering whether it can be improved *(Reflecting)*
- use a calculator to generate number patterns, using addition and subtraction *(Applying Strategies)*

Addition and Subtraction *(continued)*	**Stage 2**

Background Information

Students should be encouraged to estimate answers before attempting to solve problems in concrete or symbolic form.

There is still a need to emphasise mental computation even though students can now use a formal written method. The following formal methods may be used.

Decomposition
The following example shows a suitable layout for the decomposition method.

$$\begin{array}{r} 2\,\overset{3}{\cancel{4}}\,{}^{1}5\,6 \\ -\;1\,3\,8\,5 \\ \hline 1\,0\,7\,1 \end{array}$$

Equal Addends
For students who have a good understanding of subtraction, the equal addends algorithm may be introduced as an alternative, particularly where very large numbers are involved. There are several possible layouts of the method, of which the following is only one and not necessarily the best. The expression 'borrow and pay back' should not be used. 'Add ten ones' and 'add ten' is preferable.

$$\begin{array}{r} 3\,{}^{1}8\,{}^{1}6\,{}^{1}1\,{}^{1}2 \\ -\;{}^{1}2\,{}^{1}9\,{}^{1}8\,{}^{1}9\,3 \\ \hline 8\,7\,1\,9 \end{array}$$

When developing a formal written algorithm, it will be necessary to sequence the examples to cover the range of possibilities that include with and without trading in one or more places, and one or more zeros in the first number.

Language

Word problems requiring subtraction usually fall into two types – either 'take away' or 'comparison'. The comparison type of subtraction involves finding how many more need to be added to a group to make it equivalent to a second group, or finding the difference between two groups. Students need to be able to translate from these different language contexts into a subtraction calculation.

The word 'difference' has a specific meaning in a subtraction context.
Difficulties could arise for some students with use of the passive voice in relation to subtraction problems eg '10 take away 9' will give a different response to '10 was taken away from 9'.

Addition and Subtraction — Stage 3

NS3.2	**Key Ideas**
Selects and applies appropriate strategies for addition and subtraction with counting numbers of any size	Select and apply appropriate mental, written or calculator strategies for addition and subtraction with counting numbers of any size

Knowledge and Skills

Students learn about

- selecting and applying appropriate mental, written or calculator strategies to solve addition and subtraction problems
- using a formal written algorithm and applying place value concepts to solve addition and subtraction problems, involving counting numbers of any size
- using estimation to check solutions to addition and subtraction problems eg 1438+129 is about 1440+130
- adding numbers with different numbers of digits eg 42 000+5123+246

Working Mathematically

Students learn to

- ask 'What if' questions eg 'What happens if we subtract a larger number from a smaller number on a calculator?' *(Questioning)*
- pose problems that can be solved using counting numbers of any size and more than one operation *(Questioning)*
- explain whether an exact or approximate answer is best suited to a situation *(Communicating)*
- use a number of strategies to solve unfamiliar problems, including:
  - trial and error
  - drawing a diagram
  - working backwards
  - looking for patterns
  - using a table
  - simplifying the problem

  (Applying Strategies, Communicating)
- check solutions by using the inverse operation or a different method *(Applying Strategies, Reasoning)*
- explain how an answer was obtained for an addition or subtraction problem and justify the selected calculation method *(Communicating, Reasoning)*
- give reasons why a calculator was useful when solving a problem *(Reasoning, Applying Strategies)*
- reflect on chosen method of solution for a problem, considering whether it can be improved *(Reflecting)*

Background Information

At this Stage, mental strategies need to be continually reinforced and used to check results obtained using formal algorithms.

Students may find that their own written strategies that are based on mental strategies may be more efficient than a formal written algorithm, particularly for the case of subtraction. For example 8000 – 673 is easier to do mentally than by using either the decomposition or the equal addends methods.

Mentally:

8000 = 7999 + 1

7999 – 673 = 7326

The answer will therefore be 7326 + 1 = 7327.

This is just one way of doing this mentally: students could share possible approaches and compare them to determine the most efficient.

Decomposition Method:

$$\begin{array}{r} {}^{7}\cancel{8}\,{}^{9}\cancel{0}\,{}^{9}\cancel{0}\,{}^{1}0 \\ -\quad 6\;7\;3 \\ \hline 7\;3\;2\;7 \end{array}$$

Equal Addends Method:

$$\begin{array}{r} 8\,{}^{1}0\,{}^{1}0\,{}^{1}0 \\ -\;{}_{1}\;{}_{1}6\,{}_{1}7\;3 \\ \hline 7\;3\;2\;7 \end{array}$$

Language

Difficulties could arise for some students with use of the passive voice in relation to subtraction problems eg '10 take away 9' will give a different response to '10 was taken away from 9'.

Multiplication and Division — Stage 2

NS2.3 - Unit 1 (multiplication and division facts)

Uses mental and informal written strategies for multiplication and division

Key Ideas

Develop mental facility for number facts up to 10×10

Find multiples and squares of numbers

Knowledge and Skills

Students learn about

- counting by threes, fours, sixes, sevens, eights or nines using skip counting
- linking multiplication and division facts using groups or arrays

  eg

  • • • •
  • • • •
  • • • •

  3 groups of 4 is 12 — $3 \times 4 = 12$

  12 shared among 3 is 4 — $12 \div 3 = 4$
- using mental strategies to recall multiplication facts up to 10×10, including
  - the commutative property of multiplication eg $7 \times 9 = 9 \times 7$
  - using known facts to work out unknown facts eg $5 \times 5 = 25$ so $5 \times 6 = (5 \times 5)+5$
  - the relationship between multiplication facts eg 'the multiplication facts for 6 are double the multiplication facts for 3'
- recognising and using ÷ and $\overline{)\ }$ to indicate division
- using mental strategies to divide by a one-digit number, including
  - the inverse relationship of multiplication and division eg $63 \div 9 = 7$ because $7 \times 9 = 63$
  - recalling known division facts
  - relating to known division facts eg $36 \div 4$; halve 36 and halve again
- describing and recording methods used in solving multiplication and division problems
- listing multiples for a given number
- finding square numbers using concrete materials and diagrams

Working Mathematically

Students learn to

- recall multiplication facts up to 10×10, including zero facts *(Applying Strategies)*
- solve a variety of problems using problem-solving strategies, including:
  - trial and error
  - drawing a diagram
  - working backwards
  - looking for patterns
  - using a table *(Applying Strategies, Communicating)*
- explain why a rectangular array can be read as a division in two ways by forming vertical or horizontal groups eg $12 \div 4 = 3$ or $12 \div 3 = 4$ *(Reasoning, Communicating)*
- check the reasonableness of a solution to a problem by relating it to an original estimation *(Reasoning)*
- explain how an answer was obtained and compare own method/s of solution to a problem with those of others *(Communicating, Reflecting)*
- use multiplication and division facts in board, card and computer games *(Applying Strategies)*
- apply the inverse relationship of multiplication and division to check answers eg $63 \div 9$ is 7 because $7 \times 9 = 63$ *(Applying Strategies, Reflecting)*
- create a table or simple spreadsheet to record multiplication facts *(Applying Strategies)*
- explain why the numbers 1, 4, 9, 16, … are called square numbers *(Communicating, Reasoning, Reflecting)*

Background Information

At this Stage, the emphasis in multiplication and division is on students developing mental strategies and using their own (informal) methods for recording their strategies. Comparing their method of solution with those of others, will lead to the identification of efficient mental and written strategies.

One problem may have several acceptable methods of solution.

Linking multiplication and division is an important understanding for students at this Stage. Students should come to realise that division 'undoes' multiplication and multiplication 'undoes' division. Students should be encouraged to check the answer to a division question by multiplying their answer by the divisor. To divide, students may recall division facts or transform the division into a multiplication and use multiplication facts eg $35 \div 7$ is the same as $\square \times 7 = 35$.

Language

When beginning to build and read multiplication tables aloud, it is best to use a language pattern of words that relates back to concrete materials such as arrays.

As students become more confident with recalling multiplication number facts, they may use less language.

For example, 'seven rows (or groups) of three' becomes 'seven threes' with the 'rows of' or 'groups of' implied. This then leads to:

- one three is three
- two threes are six
- three threes are nine, and so on.

Multiplication and Division **Stage 2**

NS2.3 - Unit 2

Uses mental and informal written strategies for multiplication and division

Key Ideas

Use mental and informal written strategies for multiplying or dividing a two-digit number by a one-digit operator

Interpret remainders in division problems

Determine factors for a given number

Knowledge and Skills

Students learn about

- using mental strategies to multiply a one-digit number by a multiple of 10 (eg 3×20) by
  - repeated addition ($20+20+20 = 60$)
  - using place value concepts (3×2 tens = 6 tens = 60)
  - factoring ($3 \times 2 \times 10 = 6 \times 10 = 60$)
- using mental strategies to multiply a two-digit number by a one-digit number, including
  - using known facts eg $9 \times 10 = 90$ so $9 \times 13 = 90+9+9+9$
  - multiplying the tens and then the units eg 7×19 is $(7 \times 10) + (7 \times 9) = 70+63 = 133$
  - the relationship between multiplication facts eg 4×23 is double 23 and double again
  - factorising eg $18 \times 5 = 9 \times 2 \times 5 = 9 \times 10 = 90$
- using mental strategies to divide by a one-digit number, in problems for which answers include a remainder
  eg $29 \div 6$; if $4 \times 6 = 24$ and $5 \times 6 = 30$ the answer is 4 remainder 5
- recording remainders to division problems eg $17 \div 4 = 4$ remainder 1
- recording answers, which include a remainder, to division problems to show the connection with multiplication eg $17 = 4 \times 4 + 1$
- interpreting the remainder in the context of the word problem
- describing multiplication as the product of two or more numbers
- describing and recording methods used in solving multiplication and division problems
- determining factors for a given number eg factors of 12 are 1, 2, 3, 4, 6, 12

Working Mathematically

Students learn to

- pose and solve multiplication and division problems *(Questioning, Applying Strategies)*
- select and use mental, written and calculator strategies to solve multiplication or division problems eg 'to multiply by 12, multiply by 6 and then double' *(Applying Strategies)*
- solve a variety of problems using problem-solving strategies, including:
  - trial and error
  - drawing a diagram
  - working backwards
  - looking for patterns
  - using a table *(Applying Strategies, Communicating)*
- identify the operation/s required to solve a problem *(Applying Strategies)*
- check the reasonableness of a solution to a problem by relating it to an original estimation *(Reasoning)*
- explain how an answer was obtained and compare own method/s of solution to a problem with those of others *(Communicating, Reflecting)*
- use multiplication and division facts in board, card and computer games *(Applying Strategies)*
- apply the inverse relationship of multiplication and division to check answers eg $63 \div 9$ is 7 because $7 \times 9 = 63$ *(Applying Strategies, Reflecting)*
- explain why a remainder is obtained in answers to some division problems *(Communicating, Reasoning)*
- apply factorisation of a number to aid mental computation eg $16 \times 25 = 4 \times 4 \times 25 = 4 \times 100 = 400$ *(Applying Strategies)*

Background Information

At this Stage, the emphasis in multiplication and division is on students developing mental strategies and using their own (informal) methods for recording their strategies. Comparing their method of solution with those of others, will lead to the identification of efficient mental and written strategies.

One problem may have several acceptable methods of solution.

Students could extend their recall of number facts beyond the multiplication facts to 10×10 by also memorising multiples of numbers such as 11, 12, 15, 20 and 25.

Language

The term 'product' has a different meaning in mathematics from its everyday usage.

Multiplication and Division — Stage 3

NS3.3

Selects and applies appropriate strategies for multiplication and division

Key Ideas

Select and apply appropriate mental, written or calculator strategies for multiplication and division

Use formal written algorithms for multiplication (limit operators to two-digit numbers) and division (limit operators to single digits)

Explore prime and composite numbers

Knowledge and Skills

Students learn about

- applying appropriate mental, written or calculator strategies to solve multiplication and division problems
- recognising and using different notations to indicate division eg $25 \div 4$, $4\overline{)25}$, $\frac{25}{4}$
- recording remainders as fractions or decimals, where appropriate eg $25 \div 4 = 6\frac{1}{4}$ or 6.25
- multiplying three- and four-digit numbers by one-digit numbers using mental or written strategies

  eg 432×5

  (mental)
  $$\begin{aligned} &= 400 \times 5 + 30 \times 5 + 2 \times 5 \\ &= 2000 + 150 + 10 \\ &= 2160 \end{aligned}$$

  (written)
  $$\begin{array}{r} 432 \\ \times\ \ 5 \\ \hline 2160 \end{array}$$

- multiplying three-digit numbers by two-digit numbers using the extended form (long multiplication)

  eg
  $$\begin{array}{r} 521 \\ \times\ \ 22 \\ \hline 1042 \\ 10\,420 \\ \hline 11\,462 \end{array}$$

- dividing a number with three or more digits by a single-digit divisor using mental or written strategies

  eg $341 \div 4$

  (mental)
  $$\begin{aligned} 340 \div 4 &= 85 \\ 1 \div 4 &= \tfrac{1}{4} \\ 341 \div 4 &= 85\tfrac{1}{4} \end{aligned}$$

  (written)
  $$\begin{array}{r} 85\frac{1}{4} \\ 4\overline{)341} \end{array}$$

- using mental strategies to multiply or divide a number by 100 or a multiple of 10
- finding solutions to questions involving mixed operations eg $5 \times 4 + 7 = 27$
- determining whether a number is prime or composite by finding the number of factors eg '13 has two factors (1 and 13) and therefore is prime; 21 has more than two factors (1, 3, 7, 21) and therefore is composite'

Working Mathematically

Students learn to

- estimate answers to problems and check to justify solutions *(Applying Strategies, Reasoning)*
- select an appropriate strategy for the solution of multiplication and division problems *(Applying Strategies, Reflecting)*
- use a number of strategies to solve unfamiliar problems, including:
  - trial and error
  - drawing a diagram
  - working backwards
  - looking for patterns
  - simplifying the problem
  - using a table

  (Applying Strategies, Communicating)
- use the appropriate operation in solving problems in real-life situations *(Applying Strategies, Reflecting)*
- give a valid reason for a solution to a multiplication or division problem and check that the answer makes sense in the original situation *(Communicating, Reasoning)*
- use mathematical terminology and some conventions to explain, interpret and represent multiplication and division in a variety of ways *(Applying Strategies, Communicating)*
- use and interpret remainders in answers to division problems eg realising that the answer needs to be rounded up if the problem involves finding the number of cars needed to take 48 people to an event *(Applying Strategies, Communicating)*
- question the meaning of packaging statements when determining the best buy eg 4 toilet rolls for $2.95 or 6 toilet rolls for $3.95 *(Questioning)*
- determine that when a number is divided by a larger number a fraction which is less than 1 is the result *(Reflecting)*
- calculate averages in everyday contexts eg temperature, sport scores *(Applying Strategies)*
- explain why a prime number when modelled as an array has only one row *(Communicating, Reflecting)*

Background Information

Students could extend their recall of number facts beyond the multiplication facts to 10×10 by also memorising multiples of numbers such as 11, 12, 15, 20 and 25, and/or utilise mental strategies such as '14×6 is 10 sixes plus 4 sixes'.

One is not a prime number because it has only one factor, itself.

The simplest form of multiplication word problems relate to rates eg If four students earn $3 each, how much do they have altogether? Another type of problem is related to ratio and uses language such as 'twice as many as' and 'six times as many as'. The terms rate and ratio are not introduced at this Stage, but students need to be able to interpret these problems as requiring multiplication.

Operations with Whole Numbers	**Stage 4**
NS4.1 Recognises the properties of special groups of whole numbers and applies a range of strategies to aid computation	**Key Ideas** Explore other counting systems Investigate groups of positive whole numbers Determine and apply tests of divisibility Express a number as a product of its prime factors Find squares/related square roots; cubes/related cube roots Use index notation for positive integral indices Apply mental strategies to aid computation Divide two- or three-digit numbers by a two-digit number
Knowledge and Skills **Students learn about** • expressing a number as a product of its prime factors • using index notation to express powers of numbers (positive indices only) eg $8 = 2^3$ • using the notation for square root $\left(\sqrt{\ }\right)$ and cube root $\left(\sqrt[3]{\ }\right)$ • recognising the link between squares and square roots and cubes and cube roots eg $2^3 = 8$ and $\sqrt[3]{8} = 2$ • exploring through numerical examples that: - $(ab)^2 = a^2b^2$, eg $(2 \times 3)^2 = 2^2 \times 3^2$ - $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, eg $\sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4}$ • finding square roots and cube roots of numbers expressed as a product of their prime factors • finding square roots and cube roots of numbers using a calculator, after first estimating • identifying special groups of numbers including figurate numbers, palindromic numbers, Fibonacci numbers, numbers in Pascal's triangle • comparing the Hindu-Arabic number system with number systems from different societies past and present • determining and applying tests of divisibility • using an appropriate non-calculator method to divide two-and three-digit numbers by a two-digit number • applying a range of mental strategies to aid computation, for example - a practical understanding of associativity and commutativity eg $2 \times 7 \times 5 = 7 \times (2 \times 5) = 70$ - to multiply a number by 12, first multiply by 6 and then double the result - to multiply a number by 13, first multiply the number by ten and then add 3 times the number - to divide by 20, first halve the number and then divide by 10 - a practical understanding of the distributive law eg to multiply any number by 9 first multiply by 10 and then subtract the number	**Working Mathematically** **Students learn to** • question whether it is more appropriate to use mental strategies or a calculator to find the square root of a given number *(Questioning)* • discuss the strengths and weaknesses of different number systems *(Communicating, Reasoning)* • describe and recognise the advantages of the Hindu-Arabic number system *(Communicating, Reasoning)* • apply tests of divisibility mentally as an aid to calculation *(Applying Strategies)* • verify the various tests of divisibility (*Reasoning)*

Operations with Whole Numbers *(continued)* **Stage 4**

Background Information

This work with squares and square roots links to Pythagoras' theorem in Measurement.

Calculations with cubes and cube roots may be applied in volume problems in Measurement.

The topic of special groups of numbers links with number patterning in Patterns and Algebra.

To divide two- and three-digit numbers by a two-digit number, students may be taught the long division algorithm or, alternatively, to transform the division into a multiplication.

eg (i) $88 \div 44 = 2$ because $2 \times 44 = 88$;

(ii) $356 \div 52 = \quad$ becomes $52 \times \quad = 356$. Knowing that there are two fifties in each 100, students may try 7 so that $52 \times 7 = 364$ which is too large. Try 6, $52 \times 6 = 312$.

Answer is $6\frac{44}{52} = 6\frac{11}{13}$

Students also need to be able to express a division in the following form in order to relate multiplication and division:

$356 = 6 \times 52 + 44$

Divide by 52:

$\frac{356}{52} = 6 + \frac{44}{52} = 6\frac{44}{52} = 6\frac{11}{13}$

The square root sign signifies a positive number (or zero). Thus $\sqrt{9} = 3$ (only). However, the two numbers whose square is 9 are $\sqrt{9}$ and $-\sqrt{9}$ ie 3 and –3.

Figurate numbers include triangular numbers, square numbers, and pentagonal numbers.

The meaning (and possibly the derivation) of the 'radical sign' may provide an interesting historical perspective.

Number systems from different societies past and present could include Egyptian, Babylonian, Roman, Mayan, Aboriginal, and Papua-New Guinean. The differences to be compared may include those related to the symbols used for numbers and operations, the use of zero, the base system, place value, and notation for fractions.

The Internet is a source of information on number systems in use in other cultures and/or at other times in history.

The Chinese mathematician, Chu Shi-kie, wrote about the triangle result (which we now call Pascal's triangle) in 1303 – at least 400 years before Pascal.

Language

Note the distinction between the use of fewer/fewest for number of items and less/least for quantities eg 'There are fewer students in this class; there is less milk today.'

Words such as 'square' have more than one mathematical context eg draw a square; square three; find the square root of 9. Students may need to have these differences explained.

Words such as 'product', 'odd', 'prime' and 'power' have different meanings in mathematics from their everyday usage. This may be confusing for some students.

Integers	**Stage 4**
NS4.2 Compares, orders and calculates with integers	**Key Ideas** Perform operations with directed numbers Simplify expressions involving grouping symbols and apply order of operations

Knowledge and Skills	**Working Mathematically**
Students learn about • recognising the direction and magnitude of an integer • placing directed numbers on a number line • ordering directed numbers • interpreting different meanings (direction or operation) for the + and – signs depending on the context • adding and subtracting directed numbers • multiplying and dividing directed numbers • using grouping symbols as an operator • applying order of operations to simplify expressions • keying integers into a calculator using the +/– key • using a calculator to perform operations with integers	**Students learn to** • interpret the use of directed numbers in a real world context eg rise and fall of temperature *(Communicating)* • construct a directed number sentence to represent a real situation *(Communicating)* • apply directed numbers to calculations involving money and temperature *(Applying Strategies, Reflecting)* • use number lines in applications such as time lines and thermometer scales *(Applying Strategies, Reflecting)* • verify, using a calculator or other means, directed number operations eg subtracting a negative number is the same as adding a positive number *(Reasoning)* • question whether it is more appropriate to use mental strategies or a calculator when performing operations with integers *(Questioning)*

Background Information

Complex recording formats for directed numbers such as raised signs can be confusing. The following formats are recommended.

–2 – 3 = –5
–3 + 6 = 3
–3 + (–4) = –3 – 4 = –7
–2 – (–3) = –2 + 3 = 1
–3.25 + 6.83 = 3.58

Brahmagupta, an Indian mathematician and astronomer (*c* 598 – *c* 665 AD) is noted for the introduction of zero and negative numbers in arithmetic.

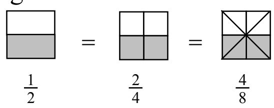
Fractions and Decimals	**Stage 2**
NS2.4 - Unit 1 Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages	**Key Ideas** Model, compare and represent fractions with denominators 2, 4 and 8, followed by fractions with denominators of 5, 10 and 100 Model, compare and represent decimals to 2 decimal places Add and subtract decimals with the same number of decimal places (to 2 decimal places) Perform calculations with money
Knowledge and Skills **Students learn about** • modelling, comparing and representing fractions with denominators 2, 4 and 8 by - modelling halves, quarters and eighths of a whole object or collection of objects - naming fractions with denominators of two, four and eight up to one whole eg $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ - comparing and ordering fractions with the same denominator eg $\frac{1}{8}$ is less than $\frac{3}{8}$ is less than $\frac{6}{8}$ - interpreting the denominator as the number of equal parts a whole has been divided into - interpreting the numerator as the number of equal fractional parts eg $\frac{3}{8}$ means 3 equal parts of 8 - comparing unit fractions by referring to the denominator or diagrams eg $\frac{1}{8}$ is less than $\frac{1}{2}$ - renaming $\frac{2}{2}, \frac{4}{4}, \frac{8}{8}$ as 1 • modelling, comparing and representing fractions with denominators 5, 10 and 100 by - extending the knowledge and skills covered above to fifths, tenths and hundredths • modelling, comparing and representing decimals to two decimal places • applying an understanding of place value to express whole numbers, tenths and hundredths as decimals • interpreting decimal notation for tenths and hundredths eg 0.1 is the same as $\frac{1}{10}$ • adding and subtracting decimals with the same number of decimal places (to 2 decimal places)	**Working Mathematically** **Students learn to** • pose questions about a collection of items eg 'Is it possible to show one-eighth of this collection of objects?' *(Questioning)* • explain why $\frac{1}{8}$ is less than $\frac{1}{4}$ eg if the cake is divided among eight people, the slices are smaller than if the cake is shared among four people *(Reasoning, Communicating)* • check whether an answer is correct by using an alternative method eg use a number line or calculator to show that $\frac{1}{2}$ is the same as 0.5 and $\frac{5}{10}$ *(Reasoning)* • interpret the everyday use of fractions and decimals, such as in advertisements *(Reflecting)* • interpret a calculator display in the context of the problem eg 2.6 means \$2.60 when using money *(Applying Strategies, Communicating)* • apply decimal knowledge to record measurements eg 123 cm = 1.23 m *(Reflecting)* • explain the relationship between fractions and decimals eg $\frac{1}{2}$ is the same as 0.5 *(Reasoning, Communicating)* • perform calculations with money *(Applying Strategies)*
Background Information At this Stage, 'commonly used fractions' refers to those with denominators 2, 4 and 8, as well as those with denominators 5, 10 and 100. Students apply their understanding of fractions with denominators 2, 4 and 8 to fractions with denominators 5, 10 and 100.	Fractions are used in different ways: - to describe equal parts of a whole - to describe equal parts of a collection of objects - to denote numbers eg $\frac{1}{2}$ is midway between 0 and 1 on the number line - as operators related to division eg dividing a number in half.
Language At this Stage it is not intended that students necessarily use the terms 'numerator' and 'denominator'. 'Decimal' is a commonly used contraction of 'decimal fraction'.	In most cases, there are differences in the meaning of fraction and ordinal terms that use the same word eg 'tenth' (fraction) has a different meaning to 'the tenth' (ordinal).

Fractions and Decimals	**Stage 2**
NS2.4 - Unit 2 Models, compares and represents commonly used fractions and decimals, adds and subtracts decimals to two decimal places, and interprets everyday percentages	**Key Ideas** Find equivalence between halves, quarters and eighths; fifths and tenths; tenths and hundredths Recognise percentages in everyday situations Relate a common percentage to a fraction or decimal

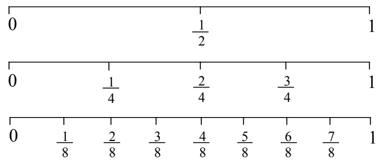
Knowledge and Skills

Students learn about

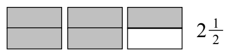
- modelling, comparing and representing fractions with denominators 2, 4 and 8 by
  - finding equivalence between halves, quarters and eighths using concrete materials and diagrams, by re-dividing the unit

    eg 

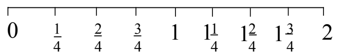
  - placing halves, quarters and eighths on a number line between 0 and 1 to further develop equivalence

    eg 

  - counting by halves and quarters eg $0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, \ldots$
  - modelling mixed numerals

    eg 

  - placing halves and quarters on a number line beyond 1

    eg 

- modelling, comparing and representing fractions with denominators 5, 10 and 100 by
  - extending the knowledge and skills covered above to fifths, tenths and hundredths
- ordering decimals with the same number of decimal places (to 2 decimal places) on a number line
- rounding a number with one or two decimal places to the nearest whole number
- recognising the number pattern formed when decimal numbers are multiplied or divided by 10 or 100
- recognising that the symbol % means 'percent'
- relating a common percentage to a fraction or decimal eg '25% means 25 out of 100 or 0.25'
- equating 10% to $\frac{1}{10}$, 25% to $\frac{1}{4}$ and 50% to $\frac{1}{2}$

Working Mathematically

Students learn to

- pose questions about a collection of items eg 'Is it possible to show one-eighth of this collection of objects?' *(Questioning)*
- check whether an answer is correct by using an alternative method eg use a number line or calculator to show that $\frac{1}{2}$ is the same as 0.5 and $\frac{5}{10}$ *(Reasoning)*
- interpret the everyday use of fractions, decimals and percentages, such as in advertisements *(Reflecting)*
- interpret a calculator display in the context of the problem eg 2.6 means $2.60 when using money *(Applying Strategies, Communicating)*
- apply decimal knowledge to record measurements eg 123 cm = 1.23 m *(Reflecting)*
- explain the relationship between fractions and decimals eg $\frac{1}{2}$ is the same as 0.5 *(Reasoning, Communicating)*
- round an answer obtained by using a calculator, to one or two decimal places *(Applying Strategies)*
- use a calculator to create patterns involving decimal numbers eg 1 ÷ 10, 2 ÷ 10, 3 ÷ 10 *(Applying Strategies)*
- perform calculations with money *(Applying Strategies)*

Background Information

Money is an application of decimals to two decimal places.

At this Stage it is not intended that students necessarily use the terms 'numerator' and 'denominator'.

Language

The decimal 1.12 is read 'one point one two' and not 'one point twelve'.

The word *cent* comes from the Latin word 'centum' meaning 'one hundred'. *Percent* means 'out of one hundred' or 'hundredths'.

Fractions and Decimals **Stage 3**

NS3.4 - Unit 1

Compares, orders and calculates with decimals, simple fractions and simple percentages

Key Ideas

Model, compare and represent commonly used fractions (those with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100)

Find equivalence between thirds, sixths and twelfths

Express a mixed numeral as an improper fraction and vice versa

Multiply and divide decimals by whole numbers in everyday contexts

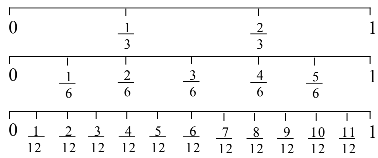
Add and subtract decimals to three decimal places

Knowledge and Skills

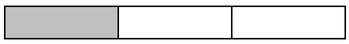
Students learn about

- modelling thirds, sixths and twelfths of a whole object or collection of objects
- placing thirds, sixths or twelfths on a number line between 0 and 1 to develop equivalence

  eg



- expressing mixed numerals as improper fractions, and vice versa, through the use of diagrams or number lines, leading to a mental strategy
- recognising that $1+\frac{1}{2}=1\frac{1}{2}$
- using written, diagram and mental strategies to subtract a unit fraction from 1 eg $1-\frac{1}{3}=\frac{2}{3}$



- using written, diagram and mental strategies to subtract a unit fraction from any whole number eg $4-\frac{1}{3}$
- adding and subtracting fractions with the same denominator eg $\frac{5}{6}+\frac{3}{6}$
- expressing thousandths as decimals
- interpreting decimal notation for thousandths
- comparing and ordering decimal numbers with three decimal places
- placing decimal numbers on a number line between 0 and 1
- adding and subtracting decimal numbers with a different number of decimal places
- multiplying and dividing decimal numbers by single digit numbers and by 10, 100 and 1000

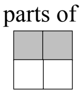
Working Mathematically

Students learn to

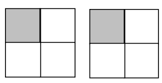
- pose and solve problems involving simple proportions eg 'If a recipe for 8 people requires 3 cups of sugar, how many cups would be needed for 4 people?' *(Questioning, Applying Strategies)*
- explain or demonstrate why two fractions are or are not equivalent *(Reasoning, Reflecting)*
- use estimation to check whether an answer is reasonable *(Applying Strategies, Reasoning)*
- interpret and explain the use of fractions, decimals and percentages in everyday contexts eg $\frac{3}{4}$ hr = 45 min *(Communicating, Reflecting)*
- apply the four operations to money problems *(Applying Strategies)*
- interpret an improper fraction in an answer *(Applying Strategies)*
- use a calculator to explore the effect of multiplying or dividing decimal numbers by multiples of ten *(Applying Strategies)*

Background Information

Fractions may be interpreted in different ways depending on the context eg two quarters ($\frac{2}{4}$) may be thought of as two equal parts of one whole that has been divided into four equal parts.



Alternatively, two quarters ($\frac{2}{4}$) may be thought of as two equal parts of two wholes that have each been divided into quarters.



$\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$

Students need to interpret a variety of word problems and translate them into mathematical diagrams and/or fraction notation. Fractions have different meanings depending on the context eg show on a diagram $\frac{3}{4}$ of a pizza; four children share three pizzas, draw a diagram to show how much each receives.

Fractions and Decimals **Stage 3**

NS3.4 - Unit 2

Compares, orders and calculates with decimals, simple fractions and simple percentages

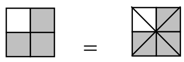
Key Ideas

Add and subtract simple fractions where one denominator is a multiple of the other
Multiply simple fractions by whole numbers
Calculate unit fractions of a number
Calculate simple percentages of quantities
Apply the four operations to money in real-life situations

Knowledge and Skills

Students learn about

- finding equivalent fractions using diagrams and number lines by re-dividing the unit

  eg 

  $\frac{3}{4}$ = $\frac{6}{8}$

- developing a mental strategy for finding equivalent fractions eg multiply or divide the numerator and the denominator by the same number
  $\frac{1}{4}=\frac{2\times1}{2\times4}=\frac{3\times1}{3\times4}=\frac{4\times1}{4\times4}$ ie $\frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}$
- reducing a fraction to its lowest equivalent form by dividing the numerator and the denominator by a common factor
- comparing and ordering fractions greater than one using strategies such as diagrams, the number line or equivalent fractions
- adding and subtracting simple fractions where one denominator is a multiple of the other
  eg $\frac{2}{3}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$
- multiplying simple fractions by whole numbers using repeated addition, leading to a rule
  eg $3\times\frac{2}{5}=\frac{2}{5}+\frac{2}{5}+\frac{2}{5}=\frac{6}{5}$ leading to $3\times\frac{2}{5}=\frac{3\times2}{5}=\frac{6}{5}$
- calculating unit fractions of a collection
  eg calculate $\frac{1}{5}$ of 30
- representing simple fractions as a decimal and as a percentage
- calculating simple percentages (10%, 20%, 25%, 50%) of quantities eg 10% of \$200 = $\frac{1}{10}$ of \$200 = \$20

Working Mathematically

Students learn to

- pose and solve problems involving simple proportions eg 'If a recipe for 8 people requires 3 cups of sugar, how many cups would be needed for 4 people?' *(Questioning, Applying Strategies)*
- explain or demonstrate why two fractions are or are not equivalent *(Reasoning, Reflecting)*
- use estimation to check whether an answer is reasonable *(Applying Strategies, Reasoning)*
- interpret and explain the use of fractions, decimals and percentages in everyday contexts eg $\frac{3}{4}$ hr = 45 min *(Communicating, Reflecting)*
- recall commonly used equivalent fractions eg 75%, 0.75, $\frac{3}{4}$ *(Communicating, Reflecting)*
- apply the four operations to money problems *(Applying Strategies)*
- use mental strategies to convert between percentages and fractions to estimate discounts *(Applying Strategies)*
- calculate prices following percentage discounts eg a 10% discount *(Applying Strategies)*
- explain how 50% of an amount could be less than 10% of another amount *(Applying Strategies, Reasoning)*
- interpret an improper fraction in an answer *(Applying Strategies)*
- use a calculator to explore and create patterns with fractions and decimals *(Applying Strategies)*

Background Information

At this Stage, 'simple fractions' refers to those with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100.

Fraction concepts are applied in other areas of mathematics eg Chance, Space and Geometry, and Measurement.

In HSIE, scale is used when reading and interpreting maps.

In Music, reading and interpreting note values links with fraction work. Semiquavers, quavers, crotchets, minims and semibreves can be compared using fractions eg a quaver is $\frac{1}{2}$ of a crotchet, and $\frac{1}{4}$ of a minim. Musicians indicate fraction values by tails on the stems of notes or by contrasting open and closed notes. Time signatures in music appear similar to fractions.

Language

In Chance, the likelihood of an outcome may be described as, for example, 'one in four'.

Students may need assistance with the subtleties of the English language when solving problems eg '10% of \$50' is not the same as '10% off \$50'.

Fractions, Decimals and Percentages **Stage 4**

NS4.3

Operates with fractions, decimals, percentages, ratios and rates

Key Ideas

Perform operations with fractions, decimals and mixed numerals

Use ratios and rates to solve problems

Knowledge and Skills

Students learn about

Fractions, Decimals and Percentages

- finding highest common factors and lowest common multiples
- finding equivalent fractions
- reducing a fraction to its lowest equivalent form
- adding and subtracting fractions using written methods
- expressing improper fractions as mixed numerals and vice versa
- adding mixed numerals
- subtracting a fraction from a whole number eg $3-\frac{2}{3}=2+1-\frac{2}{3}=2\frac{1}{3}$
- multiplying and dividing fractions and mixed numerals
- adding, subtracting, multiplying and dividing decimals (for multiplication and division, limit operators to two-digits)
- determining the effect of multiplying or dividing by a number less than one
- rounding decimals to a given number of places
- using the notation for recurring (repeating) decimals eg $0.333\ 33\ldots = 0.\dot{3}$, $0.345\ 345\ 345\ldots = 0.\dot{3}4\dot{5}$
- converting fractions to decimals (terminating and recurring) and percentages
- converting terminating decimals to fractions and percentages
- converting percentages to fractions and decimals
- calculating fractions, decimals and percentages of quantities
- increasing and decreasing a quantity by a given percentage
- interpreting and calculating percentages greater than 100% eg an increase from 6 to 18 is an increase of 200%; 150% of \$2 is \$3
- expressing profit and/or loss as a percentage of cost price or selling price
- ordering fractions, decimals and percentages
- expressing one quantity as a fraction or a percentage of another eg 15 minutes is $\frac{1}{4}$ or 25% of an hour

Working Mathematically

Students learn to

- explain multiplication of a fraction by a fraction using a diagram to illustrate the process *(Reasoning, Communicating)*
- explain why division by a fraction is equivalent to multiplication by its reciprocal *(Reasoning, Communicating)*
- choose the appropriate equivalent form for mental computation eg 10% of \$40 is $\frac{1}{10}$ of \$40 *(Applying Strategies)*
- recognise and explain incorrect operations with fractions eg explain why $\frac{2}{3}+\frac{1}{4}\neq\frac{3}{7}$ *(Applying Strategies, Reasoning, Communicating)*
- question the reasonableness of statements in the media that quote fractions, decimals or percentages eg ‘the number of children in the average family is 2.3’ *(Questioning)*
- interpret a calculator display in formulating a solution to a problem, by appropriately rounding a decimal *(Communicating, Applying Strategies)*
- recognise equivalences when calculating eg multiplication by 1.05 will increase a number/quantity by 5%, multiplication by 0.87 will decrease a number/quantity by 13% *(Applying Strategies)*
- solve a variety of real-life problems involving fractions, decimals and percentages *(Applying Strategies)*
- use a number of strategies to solve unfamiliar problems, including:
  - using a table
  - looking for patterns
  - simplifying the problem
  - drawing a diagram
  - working backwards
  - guess and refine

  (Applying Strategies, Communicating)
- interpret media and sport reports involving percentages *(Communicating)*
- evaluate best buys and special offers eg discounts *(Applying Strategies)*

Fractions, Decimals and Percentages *(continued)*	**Stage 4**
Ratio and Rates • using ratio to compare quantities of the same type • writing ratios in various forms eg $\frac{4}{6}$, 4:6, 4 to 6 • simplifying ratios eg 4:6 = 2:3, $\frac{1}{2}$:2 = 1:4, 0.3:1 = 3:10 • applying the unitary method to ratio problems • dividing a quantity in a given ratio • interpreting and calculating ratios that involve more than two numbers • calculating speed given distance and time • calculating rates from given information eg 150 kilometres travelled in 2 hours	• interpret descriptions of products that involve fractions, decimals, percentages or ratios eg on labels of packages *(Communicating)* • solve a variety of real-life problems involving ratios eg scales on maps, mixes for fuels or concrete, gear ratios *(Applying Strategies)* • solve a variety of real-life problems involving rates eg batting and bowling strike rates, telephone rates, speed, fuel consumption *(Applying Strategies)*

Background Information

Fraction concepts are applied in other areas of mathematics eg simplifying algebraic expressions, Probability, Trigonometry, and Measurement. Ratio work links with scale drawing, trigonometry and gradient of lines.

In Geography, students calculate percentage change using statistical data, and scale is used when reading and interpreting maps.

In Music, reading and interpreting note values links with fraction work. Semiquavers, quavers, crotchets, minims and semibreves can be compared using fractions eg a quaver is $\frac{1}{2}$ of a crotchet, and $\frac{1}{4}$ of a minim. Musicians indicate fraction values by tails on the stems of notes or by contrasting open and closed notes.

Time signatures in music appear similar to fractions.

In PDHPE there are opportunities for students to apply number skills eg

- when comparing time related to work, leisure and rest, students could express each as a percentage
- assessing the effect of exercise on the body by measuring the increase in pulse rate and body temperature
- calculating the height/weight ratio when analysing body composition
- conducting fitness tests such as allowing 12 minutes for a 1.6 kilometre run.

Work with ratio may be linked with the Golden Rectangle. Many windows are Golden Rectangles, as are some of the buildings in Athens such as the Parthenon. The ratio of the dimensions of the Golden Rectangle was known to the ancient Greeks:

$$\frac{\text{length}}{\text{width}} = \frac{\text{length} + \text{width}}{\text{length}}$$

The word fraction comes from the Latin *frangere* meaning ‘to break’. The earliest evidence of fractions can be traced to the Egyptian papyrus of Ahmes (about 1650 BC). In the seventh century AD the method of writing fractions as we write them now was invented in India, but without the fraction bar (vinculum), which was introduced by the Arabs. Fractions were widely in use by the 12th century.

The word ‘cent’ comes from the Latin word ‘centum’ meaning ‘one hundred’. *Percent* means ‘out of one hundred’ or ‘hundredths’.

One cent and two cent coins were withdrawn by the Australian Government in 1990. Prices can still be expressed in one-cent increments but the final bill is rounded to the nearest five cents. In this context, rounding is different to normal conventions in that totals ending in 3, 4, 6, and 7 are rounded to the nearest 5 cents, and totals ending in 8, 9, 1, and 2 are rounded to the nearest 0 cents.

Language

Students may need assistance with the subtleties of the English language when solving word problems eg ‘$\frac{1}{10}$ of \$50’ is not the same as ‘$\frac{1}{10}$ off \$50’.

Students may wrongly interpret words giving a mathematical instruction (eg estimate, multiply, simplify) to just mean ‘get the answer’.

Rational Numbers	**Stage 5.1**
NS5.1.1 Applies index laws to simplify and evaluate arithmetic expressions and uses scientific notation to write large and small numbers	**Key Ideas** Define and use zero index and negative integral indices Develop the index laws arithmetically Use index notation for square and cube roots Express numbers in scientific notation (positive and negative powers of 10)

Knowledge and Skills

Students learn about

- describing numbers written in index form using terms such as base, power, index, exponent
- evaluating numbers expressed as powers of positive whole numbers
- establishing the meaning of the zero index and negative indices eg by patterns

3^2	3^1	3^0	3^{-1}	3^{-2}
9	3	1	$\frac{1}{3}$	$\frac{1}{9}=\frac{1}{3^2}$

- writing reciprocals of powers using negative indices
  eg $3^{-4}=\frac{1}{3^4}=\frac{1}{81}$
- translating numbers to index form (integral indices) and vice versa
- developing index laws arithmetically by expressing each term in expanded form
  eg $3^2\times3^4=(3\times3)\times(3\times3\times3\times3)=3^{2+4}=3^6$
  $3^5\div3^2=\frac{3\times3\times3\times3\times3}{3\times3}=3^{5-2}=3^3$
  $(3^2)^4=(3\times3)\times(3\times3)\times(3\times3)\times(3\times3)=3^{2\times4}=3^8$
- using index laws to simplify expressions
- using index laws to define fractional indices for square and cube roots
  eg $\left(\sqrt{9}\right)^2=9$ and $\left(9^{\frac{1}{2}}\right)^2=9$, hence $\sqrt{9}=9^{\frac{1}{2}}$
- writing square roots and cube roots in index form
  eg $8^{\frac{1}{3}}=\sqrt[3]{8}=2$
- recognising the need for a notation to express very large or very small numbers
- expressing numbers in scientific notation
- entering and reading scientific notation on a calculator
- using index laws to make order of magnitude checks for numbers in scientific notation
  eg $\left(3.12\times10^4\right)\times\left(4.2\times10^6\right)\approx12\times10^{10}\approx1.2\times10^{11}$
- converting numbers expressed in scientific notation to decimal form
- ordering numbers expressed in scientific notation

Working Mathematically

Students learn to

- solve numerical problems involving indices *(Applying Strategies)*
- explain the incorrect use of index laws
  eg why $3^2\times3^4\neq9^6$ *(Communicating, Reasoning)*
- verify the index laws by using a calculator
  eg to compare the values of $\left(\sqrt{5}\right)^2$, $\left(5^{\frac{1}{2}}\right)^2$ and 5
  (Reasoning)
- communicate and interpret technical information using scientific notation *(Communicating)*
- explain the difference between numerical expressions such as 2×10^4 and 2^4, particularly with reference to calculator displays *(Communicating, Reasoning)*
- solve problems involving scientific notation *(Applying Strategies)*

Rational Numbers *(continued)* **Stage 5.1**

Background Information

This topic links to simplifying algebraic expressions using the index laws and with the use of scientific notation in Science.

At this Stage the use of index notation for square roots and cube roots is mainly to facilitate calculator use.

Language

There is a need when teaching index notation to pay particular attention to how these numbers are said eg 3^4 is 'three to the power of four' or 'three to the fourth', 4^3 is 'four cubed' or 'four to the power of three', and 2.34×10^{-5} (scientific notation) is 'two point three four times ten to the power of negative five'.

Rational Numbers **Stage 5.2**

NS5.2.1

Rounds decimals to a specified number of significant figures, expresses recurring decimals in fraction form and converts rates from one set of units to another

Key Ideas

Round numbers to a specified number of significant figures

Express recurring decimals as fractions

Convert rates from one set of units to another

Knowledge and Skills

Students learn about

- identifying significant figures
- rounding numbers to a specified number of significant figures
- using the language of estimation appropriately, including:
  - rounding
  - approximate
  - level of accuracy
- using symbols for approximation eg $\approx$
- determining the effect of truncating or rounding during calculations on the accuracy of the results
- writing recurring decimals in fraction form using calculator and non-calculator methods

  eg $0.\dot{2}$, $0.\dot{2}\dot{3}$, $0.2\dot{3}$
- converting rates from one set of units to another eg km/h to m/s, interest rate of 6% per annum is 0.5% per month

Working Mathematically

Students learn to

- recognise that calculators show approximations to recurring decimals

  eg $\frac{2}{3}$ displayed as 0.666667 *(Communicating)*
- justify that $0.\dot{9} = 1$ *(Reasoning)*
- decide on an appropriate level of accuracy for results of calculations *(Applying Strategies)*
- assess the effect of truncating or rounding during calculations on the accuracy of the results *(Reasoning)*
- appreciate the importance of the number of significant figures in a given measurement *(Communicating)*
- use an appropriate level of accuracy for a given situation or problem solution *(Applying Strategies)*
- solve problems involving rates *(Applying Strategies)*

Background Information

This topic links with work in Measurement.

§ Real Numbers **Stage 5.3**

§ NS5.3.1

Performs operations with surds and indices

Key Ideas

Define the system of real numbers distinguishing between rational and irrational numbers

Perform operations with surds

Use integers and fractions for index notation

Convert between surd and index form

Knowledge and Skills

Students learn about

- defining a rational number:

  A rational number is the ratio $\frac{a}{b}$ *of two integers where* $b \neq 0$.
- distinguishing between rational and irrational numbers
- using a pair of compasses and a straight edge to construct simple rationals and surds on the number line
- defining real numbers:

  Real numbers are represented by points on the number line.

  Irrational numbers are real numbers that are not rational.
- demonstrating that $\sqrt{x}$ is undefined for $x < 0$, $\sqrt{x} = 0$ for $x = 0$, and $\sqrt{x}$ is the positive square root of x when $x > 0$
- using the following results for $x, y > 0$:

$$\left(\sqrt{x}\right)^2 = x = \sqrt{x^2}$$
$$\sqrt{xy} = \sqrt{x}.\sqrt{y}$$
$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

- using the four operations of addition, subtraction, multiplication and division to simplify expressions involving surds
- expanding expressions involving surds such as

$$\left(\sqrt{3}+\sqrt{5}\right)^2 \text{ or } \left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)$$

- rationalising the denominators of surds of the form

$$\frac{a\sqrt{b}}{c\sqrt{d}}$$

- using the index laws to demonstrate the reasonableness of the definitions for fractional indices

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$
$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

- translating expressions in surd form to expressions in index form and vice versa
- evaluating numerical expressions involving fractional indices eg $27^{\frac{2}{3}}$

Working Mathematically

Students learn to

- explain why all integers and recurring decimals are rational numbers *(Communicating, Reasoning)*
- explain why rational numbers can be expressed in decimal form *(Communicating, Reasoning)*
- demonstrate that not all real numbers are rational *(Communicating, Applying Strategies, Reasoning)*
- solve numerical problems involving surds and/or fractional indices *(Applying Strategies)*
- explain why a particular sentence is incorrect

  eg $\sqrt{3}+\sqrt{5} \neq \sqrt{8}$ *(Communicating, Reasoning)*
- prove some general properties of numbers such as
  - the sum of two odd integers is even
  - the product of an odd and even integer is even
  - the sum of 3 consecutive integers is divisible by 3

  (Reasoning)

§ Real Numbers *(continued)*	**Stage 5.3**
• using the $x^{\frac{1}{y}}$ key on a calculator • evaluating a fraction raised to the power of –1, leading to $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$	

Background Information

Operations with surds are applied when simplifying algebraic expressions.

This topic can be linked with graphing $y = \sqrt{x}$.

Some students may enjoy a demonstration of the proof, by contradiction, that $\sqrt{2}$ is irrational.

Early Greek mathematicians believed that the length of any line would always be given by a rational number. This was proved to be false when Pythagoras and his followers found the length of the hypotenuse of an isosceles right-angled triangle with side length one unit.

Having expanded binomial products and rationalised denominators of surds of the form $\frac{a\sqrt{b}}{c\sqrt{d}}$, students could rationalise denominators of surds with binomial denominators.

Language

There is a need to emphasise how to read and articulate surds and fractional indices
eg $\sqrt{x}$ is 'the square root of x' or 'root x'.

Consumer Arithmetic	**Stage 5.1**
NS5.1.2 Solves consumer arithmetic problems involving earning and spending money	**Key Ideas** Solve simple consumer problems including those involving earning and spending money Calculate simple interest and find compound interest using a calculator and tables of values

Knowledge and Skills

Students learn about

- calculating earnings for various time periods from different sources, including:
  - wage
  - salary
  - commission
  - piecework
  - overtime
  - bonuses
  - holiday loadings
  - interest on investments
- calculating income earned in casual and part-time jobs, considering agreed rates and special rates for Sundays and public holidays
- calculating weekly, fortnightly, monthly and yearly incomes
- calculating net earnings considering deductions such as taxation and superannuation
- calculating simple interest using the formula

  $$I = PRT \text{ where } R = \frac{r}{100}$$

  where I is the interest, P the principal, R the annual interest rate and T the number of years
- applying the simple interest formula to problems related to investing money at simple interest rates
- calculating compound interest for two or three years by repeated multiplication using a calculator eg a rate of 5% per annum leads to repeated multiplication by 1.05
- calculating compound interest on investments using a table
- calculating and comparing the cost of purchasing goods using:
  - cash
  - credit card
  - lay-by
  - deferred payment
  - buying on terms
  - loans
- calculating a 'best buy'

Working Mathematically

Students learn to

- read and interpret pay slips from part-time jobs when questioning the details of their own employment *(Questioning, Communicating)*
- prepare a budget for a given income, considering such expenses as rent, food, transport etc *(Applying Strategies)*
- interpret the different ways of indicating wages or salary in newspaper 'positions vacant' advertisements eg $20K *(Communicating)*
- compare employment conditions for different careers where information is gathered from a variety of mediums including the Internet eg employment rates, payment *(Applying Strategies)*
- compare simple interest with compound interest in practical situations eg loans *(Applying Strategies)*
- interpret spreadsheets or tables when comparing simple interest and compound interest on an investment over various time periods *(Applying Strategies, Communicating)*
- realise the total cost and/or hidden costs involved in some types of purchase arrangements *(Applying Strategies)*
- make informed decisions related to purchases eg determining the best mobile phone plan for a given situation *(Applying Strategies)*
- interpret the GST on receipts *(Communicating)*

Background Information

It is not intended at this Stage for students to use the formula for compound interest. Internet sites may be used to find commercial rates for home loans and 'home loan calculators'.

The abbreviation K comes from the Greek word *khilioi* meaning thousand. It is used in many job advertisements eg a salary of $70K.

Consumer Arithmetic	**Stage 5.2**
NS5.2.2 Solves consumer arithmetic problems involving compound interest, depreciation, and successive discounts	**Key Ideas** Use compound interest formula Solve consumer arithmetic problems involving compound interest, depreciation and successive discounts
Knowledge and Skills **Students learn about** • calculating the result of successive discounts • calculating compound interest on investments and loans using repetition of the formula for simple interest • determining and using the formula for compound interest, $A = P(1+R)^n$, where A is the total amount, P is the principal, R is the interest rate per period as a decimal and n is the number of periods • using the compound interest formula to calculate depreciation • comparing the cost of loans using flat and reducible interest for a small number of repayment periods	**Working Mathematically** **Students learn to** • solve problems involving discounts and compound interest *(Applying Strategies)* • explain why, for example, a discount of 10% following a discount of 15% is not the same as a discount of 25% *(Applying Strategies, Communicating, Reasoning)* • question the advantages of interest being calculated on the basis of different time periods eg monthly rather than yearly *(Questioning, Applying Strategies)* • analyse promotional and advertising material related to finance that is collected from a variety of sources including the Internet eg loan repayments on home purchases *(Applying Strategies, Reasoning, Communicating)* • use a ‘guess and refine’ strategy when investigating unfamiliar problems *(Applying Strategies)*

Background Information

The work in this topic links with Commerce.

Internet sites may be used to find commercial rates for home loans and ‘home loan calculators’.

Chance	**Stage 2**
NS2.5 Describes and compares chance events in social and experimental contexts	**Key Ideas** Explore all possible outcomes in a simple chance situation Conduct simple chance experiments Collect data and compare likelihood of events in different contexts
Knowledge and Skills **Students learn about** • listing all the possible outcomes in a simple chance situation eg 'heads', 'tails' if a coin is tossed • distinguishing between certain and uncertain events • comparing familiar events and describing them as being equally likely or more or less likely to occur • predicting and recording all possible outcomes in a simple chance experiment eg randomly selecting three pegs from a bag containing an equal number of pegs of two colours • ordering events from least likely to most likely eg 'having ten children away sick on the one day is less likely than having one or two away' • using the language of chance in everyday contexts eg a fifty-fifty chance, a one in two chance • predicting and recording all possible combinations eg the number of possible outfits arising from three different t-shirts and two different pairs of shorts • conducting simple experiments with random generators such as coins, dice or spinners to inform discussion about the likelihood of outcomes eg roll a die fifty times, keep a tally and graph the results	**Working Mathematically** **Students learn to** • discuss the 'fairness' of simple games involving chance *(Communicating)* • compare the likelihood of outcomes in a simple chance experiment eg from a collection of 27 red, 10 blue and 13 yellow marbles, name red as being the colour most likely to be drawn out *(Reasoning)* • apply an understanding of equally likely outcomes in situations involving random generators such as dice, coins and spinners *(Reflecting)* • make statements that acknowledge 'randomness' in a situation eg 'the spinner could stop on any colour' *(Communicating, Reflecting)* • explain the differences between expected results and actual results in a simple chance experiment *(Communicating, Reflecting)*
Background Information When a fair coin is tossed, theoretically there is an equal chance of a head or tail. If the coin is tossed and there are five heads in a row there is still an equal chance of a head or tail on the next toss, since each toss is an independent event.	

Chance	**Stage 3**
NS3.5 Orders the likelihood of simple events on a number line from zero to one	**Key Ideas** Assign numerical values to the likelihood of simple events occurring Order the likelihood of simple events on a number line from 0 to 1
Knowledge and Skills **Students learn about** • using data to order chance events from least likely to most likely eg roll two dice twenty times and order the results according to how many times each total is obtained • ordering commonly used ‘chance words’ on a number line between zero (impossible) and one (certain) eg ‘equal chance’ would be placed at 0.5 • using knowledge of equivalent fractions and percentages to assign a numerical value to the likelihood of a simple event occurring eg there is a five in ten, $\frac{5}{10}$, 50% or one in two chance of this happening • describing the likelihood of events as being more or less than a half (50% or 0.5) and ordering the events on a number line • using samples to make predictions about a larger ‘population’ from which the sample comes eg predicting the proportion of cubes of each colour in a bag after taking out a sample of the cubes	**Working Mathematically** **Students learn to** • predict and discuss whether everyday events are more or less likely to happen or whether they have an equal chance of occurring *(Applying Strategies, Communicating)* • assign numerical values to the likelihood of simple events occurring in real-life contexts eg ‘My football team has a fifty-fifty chance of winning the game.’ *(Applying Strategies, Reflecting)* • describe the likelihood of an event occurring as being more or less than half *(Communicating, Reflecting)* • question whether their prediction about a larger population, from which a sample comes, would be the same if a different sample was used eg ‘Would the results be the same if a different class was surveyed?’ *(Questioning, Reflecting)* • design a spinner or label a die so that a particular outcome is more likely than another *(Applying Strategies)*
Background Information Students will need some prior experience ordering decimal fractions (tenths) on a number line from zero to one.	Chance events can be ordered on a scale from zero to one. A chance of zero describes an event that is impossible. A chance of one describes an event that is certain. Therefore, events with an equal chance of occurring can be described as having a chance of 0.5. Other expressions of chance fall between zero and one eg ‘unlikely’ will take a numerical value somewhere between 0 and 0.5.

Probability	**Stage 4**
NS4.4 Solves probability problems involving simple events	**Key Ideas** Determine the probability of simple events Solve simple probability problems Recognise complementary events
Knowledge and Skills **Students learn about** • listing all possible outcomes of a simple event • using the term 'sample space' to denote all possible outcomes eg for tossing a fair die, the sample space is 1, 2, 3, 4, 5, 6 • assigning probabilities to simple events by reasoning about equally likely outcomes eg the probability of a 5 resulting from the throw of a fair die is $\frac{1}{6}$ • expressing the probability of a particular outcome as a fraction between 0 and 1 • assigning a probability of zero to events that are impossible and a probability of one to events that are certain • recognising that the sum of the probabilities of all possible outcomes of a simple event is 1 • identifying the complement of an event eg 'The complement of drawing a red card from a deck of cards is drawing a black card.' • finding the probability of a complementary event	**Working Mathematically** **Students learn to** • solve simple probability problems arising in games *(Applying Strategies)* • use language associated with chance events appropriately *(Communicating)* • evaluate media statements involving probability *(Applying Strategies, Communicating)* • interpret and use probabilities expressed as percentages or decimals *(Applying Strategies, Communicating)* • explain the meaning of a probability of 0, $\frac{1}{2}$ and 1 in a given situation *(Communicating, Reasoning)*

Background Information

A simple event is an event in which each possible outcome is equally likely eg tossing a fair die.

Probability	**Stage 5.1**
NS5.1.3 Determines relative frequencies and theoretical probabilities	**Key Ideas** Determine relative frequencies to estimate probabilities Determine theoretical probabilities
Knowledge and Skills **Students learn about** • repeating an experiment a number of times to determine the relative frequency of an event • estimating the probability of an event from experimental data using relative frequencies • expressing the probability of an event A given a finite number of equally likely outcomes as $P(A) = \dfrac{\text{number of favourable outcomes}}{n}$ where n is the total number of outcomes in the sample space • using the formula to calculate probabilities for simple events • simulating probability experiments using random number generators	**Working Mathematically** **Students learn to** • recognise and explain differences between relative frequency and theoretical probability in a simple experiment *(Communicating, Reasoning)* • apply relative frequency to predict future experimental outcomes *(Applying Strategies)* • design a device to produce a specified relative frequency eg a four-coloured circular spinner *(Applying Strategies)* • recognise that probability estimates become more stable as the number of trials increases *(Reasoning)* • recognise randomness in chance situations *(Communicating)* • apply the formula for calculating probabilities to problems related to card, dice and other games *(Applying Strategies)*

Background Information

This topic links with relative frequency in the Data strand.

Software programs could be used for simulation experiments to demonstrate that the relative frequency gets closer and closer to the theoretical probability as the number of trials increases.

Students may not appreciate the significance of a simulation experiment eg they may not transfer random number generator results for tossing a die to the situation of actually tossing a die a number of times.

Probability	**Stage 5.3**
NS5.3.2 Solves probability problems involving compound events	**Key Ideas** Solve probability problems including two-stage and compound events

Knowledge and Skills

Students learn about

- distinguishing informally between dependent and independent events
- sampling with and without replacement in two-stage experiments eg drawing two counters from a bag containing 3 blue, 4 red and 1 white counter
- analysing two-stage events through constructing organised lists, tables and/or tree diagrams
- solving two-stage probability problems including instances of sampling with and without replacement
- finding probabilities of compound events using organised lists, tables or diagrams
  eg the table below represents data collected on 300 athletes and compares height with weight – what is the probability of choosing a light, short athlete from the population represented in the table?

	Heavy	Light	
Tall	70	20	90
Short	50	160	210
	120	180	300

Working Mathematically

Students learn to

- critically evaluate statements on chance and probability appearing in the media and/or in other subjects (*Reasoning*)
- evaluate the likelihood of winning a prize in lotteries and other competitions
  (Applying Strategies, Reasoning)
- evaluate games for fairness
  (Applying Strategies, Reasoning)
- identify common misconceptions related to chance events eg if you get four tails in a row when tossing a coin, there is a greater chance that the next outcome is a head *(Applying Strategies)*
- recognise the use of probability by governments and companies eg in demography, insurance, planning for roads *(Reflecting)*

Background Information

Venn diagrams may be used as a tool in solving some probability problems.

The mathematical analysis of probability was prompted by the French gentleman gambler, the Chevalier de Meré. Over the years, the Chevalier had consistently won money betting on at least one six in four rolls of a die. He felt that he should also win betting on at least one double six in 24 rolls of two dice, but in fact regularly lost.

In 1654 he asked his mathematician friend Pascal to explain why. This question led to a famous correspondence between Pascal and the renowned mathematician Fermat. The Chevalier's change of fortune is explained by the fact that the chance of at least one six in four rolls of a die is 51.8%, while the chance of at least one double six in 24 rolls of two dice is 49.1%.

9.3 Patterns and Algebra

The Patterns and Algebra strand extends from Early Stage 1 to Stage 5.3. In the early Stages students explore number and pre-algebra concepts by pattern making, and discussing, generalising and recording their observations.

A concrete approach to Algebra is continued when students apply their understanding of the number system to the algebraic symbol system. Students develop an understanding of the notion of a variable, establish the equivalence of expressions, apply algebraic conventions and graph relationships on the number plane.

Students recognise that graphing is a powerful tool that enables algebraic relationships to be visualised. By the end of Stage 5.3 students have the opportunity to develop knowledge of the shapes of graphs of different relationships and understand the effects on graphs of making a change to the relationship eg adding a constant. Technology is a useful tool for students to use when graphing and comparing graphs of relationships.

Algebra has strong links with all the other strands in the syllabus, particularly when situations are to be generalised symbolically.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Stage 2 to Stage 3 in one substrand. The Stage 4 content is presented in the topics Number Patterns, Algebraic Techniques and Linear Relationships. Stage 5.1 contains Algebraic Techniques and Coordinate Geometry. Stage 5.2 contains Algebraic Techniques, Coordinate Geometry, and Graphs of Physical Phenomena. Stage 5.3 builds on each of the Stage 5.2 topics and includes Curve Sketching and Polynomials and Functions and Logarithms as optional topics.

Summary of Patterns and Algebra Outcomes for Stages 2 to 5 with page references

Patterns and Algebra

PAS2.1 Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values (p 79)

PAS3.1a Records, analyses and describes geometric and number patterns that involve one operation using tables and words (p 80)

PAS3.1b Constructs, verifies and completes number sentences involving the four operations with a variety of numbers (p 81)

Algebraic Techniques

PAS4.1 Uses letters to represent numbers and translates between words and algebraic symbols (p 82)

Number Patterns

PAS4.2 Creates, records, analyses and generalises number patterns using words and algebraic symbols in a variety of ways (p 83)

Algebraic Techniques

PAS4.3 Uses the algebraic symbol system to simplify, expand and factorise simple algebraic expressions (p 85)

PAS4.4 Uses algebraic techniques to solve linear equations and simple inequalities (p 86)

PAS5.1.1 Applies the index laws to simplify algebraic expressions (p 87)

PAS5.2.1 Simplifies, expands and factorises algebraic expressions involving fractions and negative and fractional indices (p 88)

PAS5.2.2 Solves linear and simple quadratic equations, solves linear inequalities and solves simultaneous equations using graphical and analytical methods (p 90)

§PAS5.3.1 Uses algebraic techniques to simplify expressions, expand binomial products and factorise quadratic expressions (p 92)

§PAS5.3.2 Solves linear, quadratic and simultaneous equations, solves and graphs inequalities, and rearranges literal equations (p 94)

Linear Relationships

PAS4.5 Graphs and interprets linear relationships on the number plane (p 96)

Coordinate Geometry

PAS5.1.2 Determines the midpoint, length and gradient of an interval joining two points on the number plane and graphs linear and simple non-linear relationships from equations (p 97)

PAS5.2.3 Uses formulae to find midpoint, distance and gradient and applies the gradient/intercept form to interpret and graph straight lines (p 99)

PAS5.2.4 Draws and interprets graphs including simple parabolas and hyperbolas (p 101)

§PAS5.3.3 Uses various standard forms of the equation of a straight line and graphs regions on the number plane (p 102)

§PAS5.3.4 Draws and interprets a variety of graphs including parabolas, cubics, exponentials and circles and applies coordinate geometry techniques to solve problems (p 103)

Graphs of Physical Phenomena

PAS5.2.5 Draws and interprets graphs of physical phenomena (p 105)

PAS5.3.5 Analyses and describes graphs of physical phenomena (p 106)

 # Curve Sketching and Polynomials

#PAS5.3.6 Uses a variety of techniques to sketch a range of curves and describes the features of curves from the equation (p 107)

#PAS5.3.7 Recognises, describes and sketches polynomials, and applies the factor and remainder theorems to solve problems (p 108)

 # Functions and Logarithms

PAS5.3.8 Describes, interprets and sketches functions and uses the definition of a logarithm to establish and apply the laws of logarithms (p 109)

(# optional topics as further preparation for the Mathematics Extension courses in Stage 6)

(§ recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course)

Patterns and Algebra **Stage 2**

PAS2.1

Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

Key Ideas

Generate, describe and record number patterns using a variety of strategies
Build number relationships by relating multiplication and division facts to at least 10×10
Complete simple number sentences by calculating the value of a missing number

Knowledge and Skills

Students learn about

Number Patterns

- identifying and describing patterns when counting forwards or backwards by threes, fours, sixes, sevens, eights or nines
- creating, with materials or a calculator, a variety of patterns using whole numbers, fractions or decimals
  eg $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \ldots$
  2.2, 2.0, 1.8, 1.6, …
- finding a higher term in a number pattern given the first five terms eg determine the 10^{th} term given a number pattern beginning with 4, 8, 12, 16, 20, …
- describing a simple number pattern in words

Number Relationships

- using the equals sign to record equivalent number relationships and to mean 'is the same as' rather than as an indication to perform an operation
  eg $4 \times 3 = 6 \times 2$
- building the multiplication facts to at least 10×10 by recognising and describing patterns and applying the commutative property eg $6 \times 4 = 4 \times 6$
- forming arrays using materials to demonstrate multiplication patterns and relationships
  eg $3 \times 5 = 15$ • • • • •
  • • • • •
  • • • • •
- relating multiplication and division facts
  eg $6 \times 4 = 24$; so $24 \div 4 = 6$ and $24 \div 6 = 4$
- applying the associative property of addition and multiplication to aid mental computation
  eg $2 + 3 + 8 = 2 + 8 + 3$, $2 \times 3 \times 5 = 2 \times 5 \times 3$
- completing number sentences involving one operation by calculating missing values
  eg find $\square$ so that $5 + \square = 13$; find $\square$ so that $28 = \square \times 7$
- transforming a division calculation into a multiplication problem eg find $\square$ so that $30 \div 6 = \square$ becomes find $\square$ so that $\square \times 6 = 30$.

Working Mathematically

Students learn to

- pose problems based on number patterns *(Questioning)*
- solve a variety of problems using problem-solving strategies, including:
  - trial and error
  - drawing a diagram
  - working backwards
  - looking for patterns
  - using a table *(Applying Strategies, Communicating)*
- ask questions about how number patterns have been created and how they can be continued *(Questioning)*
- generate a variety of number patterns that increase or decrease and record them in more than one way *(Applying Strategies, Communicating)*
- generate number patterns using the process of repeatedly adding the same number on a calculator *(Communicating)*
- model and then record number patterns using diagrams, words or symbols *(Communicating)*

- check solutions to missing elements in patterns by repeating the process *(Reasoning)*
- play 'guess my rule' games eg 1, 4, 7: what is the rule? *(Applying Strategies)*
- describe what has been learnt from creating patterns, making connections with addition facts and multiplication facts *(Communicating, Reflecting)*
- explain the relationship between multiplication facts eg explain how the 3 and 6 times tables are related *(Reflecting)*
- make generalisations about numbers and number relationships eg 'It doesn't matter what order you multiply two numbers because the answer is always the same.' *(Reflecting)*
- check number sentences to determine if they are true or false, and, if false, explain why *(Applying Strategies, Reasoning)*
- justify a solution to a number sentence *(Reasoning)*
- use inverse operations to complete number sentences *(Applying Strategies)*
- describe strategies for completing simple number sentences *(Communicating)*

Patterns and Algebra **Stage 3**

PAS3.1a

Records, analyses and describes geometric and number patterns that involve one operation using tables and words

Key Ideas

Build simple geometric patterns involving multiples

Complete a table of values for geometric and number patterns

Describe a pattern in words in more than one way

Knowledge and Skills

Students learn about

- working through a process of building a simple geometric pattern involving multiples, completing a table of values, and describing the pattern in words. This process includes the following steps:
  - building a simple geometric pattern using materials eg **Δ, ΔΔ, ΔΔΔ, ΔΔΔΔ, …**
  - completing a table of values for the geometric pattern

eg

Number of Triangles	1	2	3	4	5	6
Number of Sides	3	6	9	12	–	–

  - describing the number pattern in a variety of ways and recording descriptions using words eg ‘It looks like the three times tables.’
  - determining a rule to describe the pattern from the table eg ‘You multiply the top number by three to get the bottom number.’
  - using the rule to calculate the corresponding value for a larger number
- working through a process of identifying a simple number pattern involving only one operation, completing a table of values, and describing the pattern in words. This process includes the following steps:
  - completing a table of values for a number pattern involving one operation (including patterns that decrease)

eg

First Number	1	2	3	4	5	6
Second Number	4	5	6	7	–	–

  - describing the pattern in a variety of ways and recording descriptions using words
  - determining a rule to describe the pattern from the table
  - using the rule to calculate the corresponding value for a larger number

Working Mathematically

Students learn to

- ask questions about how number patterns have been created and how they can be continued *(Questioning)*
- interpret sentences written by peers and teachers that accurately describe geometric and number patterns *(Applying Strategies)*
- identify patterns in data displayed in a spreadsheet *(Applying Strategies)*
- generate a variety of number patterns that increase or decrease and record in more than one way *(Applying Strategies, Communicating)*
- model and then record number patterns using materials, diagrams, words or symbols *(Applying Strategies)*
- use a number of strategies to solve unfamiliar problems, including:
  - trial and error
  - drawing a diagram
  - working backwards
  - looking for patterns
  - using a table *(Applying Strategies, Communicating)*
- check solutions to missing elements in patterns by repeating the process *(Reasoning)*
- describe what has been learnt from creating patterns, making connections with number facts and number properties *(Communicating, Reflecting)*
- make generalisations about numbers and number relationships eg ‘If you add a number and then subtract the same number, the result is the number you started with.’ *(Reflecting)*
- play ‘guess my rule’ games *(Applying Strategies)*
- describe and justify the choice of a particular rule for the values in a table *(Communicating, Reasoning)*

Background Information

This topic involves *algebra without symbols*. Symbols should not be introduced until the students have had considerable experience describing patterns in their own words.

Students should be given opportunities to discover and create patterns and to describe, in their own words, relationships contained in those patterns.

Language

At this Stage, students should be encouraged to use their own words to describe number patterns. Patterns can usually be described in more than one way and it is important for students to hear how other students describe the same pattern.

Students’ descriptions of number patterns can then become more sophisticated as they experience a variety of ways of describing the same pattern. The teacher could begin to model the use of more appropriate mathematical language to encourage this development.

Patterns and Algebra **Stage 3**

PAS3.1b

Constructs, verifies and completes number sentences involving the four operations with a variety of numbers

Key Ideas

Construct, verify and complete number sentences involving the four operations with a variety of numbers

Knowledge and Skills

Students learn about

- completing number sentences that involve more than one operation by calculating missing values
  eg $5 + \square = 12 - 4$
- completing number sentences involving fractions or decimals eg $7 \times \square = 7.7$
- constructing a number sentence to match a problem that is presented in words and requires finding an unknown eg 'I am thinking of a number so that when I double it and add 5 the answer is 13. What is the number?'
- checking solutions to number sentences by substituting the solution into the original question
- identifying and using inverse operations to assist with the solution of number sentences
  eg Find $\square$ so that $125 \div 5 = \square$ becomes find $\square$ so that $\square \times 5 = 125$.

Working Mathematically

Students learn to

- describe strategies for completing simple number sentences and justify solutions *(Communicating)*
- describe how inverse operations can be used to solve a number sentence *(Applying Strategies, Communicating)*

Background Information

Students will typically use trial-and-error methods to find solutions to number sentences. They need to be encouraged to work backwards and to describe the processes using inverse operations. The inclusion of sentences that do not have whole number solutions will aid this process.

Algebraic Techniques	**Stage 4**
PAS4.1 Uses letters to represent numbers and translates between words and algebraic symbols	**Key Ideas** Use letters to represent numbers Translate between words and algebraic symbols and between algebraic symbols and words Recognise and use simple equivalent algebraic expressions

Knowledge and Skills

Students learn about

- using letters to represent numbers and developing the notion that a letter is used to represent a variable
- using concrete materials such as cups and counters to model:
  - expressions that involve a variable and a variable plus a constant eg $a, a+1$
  - expressions that involve a variable multiplied by a constant eg $2a, 3a$
  - sums and products eg $2a+1,\ 2(a+1)$
  - equivalent expressions such as $x+x+y+y+y = 2x+2y+y = 2(x+y)+y$
  - and to assist with simplifying expressions, such as
    $$\begin{aligned}(a+2)+(2a+3) &= (a+2a)+(2+3)\\ &= 3a+5\end{aligned}$$
- recognising and using equivalent algebraic expressions
  eg
  $$y+y+y+y = 4y$$
  $$w\times w = w^2$$
  $$a\times b = ab$$
  $$a\div b = \frac{a}{b}$$
- translating between words and algebraic symbols and between algebraic symbols and words

Working Mathematically

Students learn to

- generate a variety of equivalent expressions that represent a particular situation or problem *(Applying Strategies)*
- describe relationships between the algebraic symbol system and number properties *(Reflecting, Communicating)*
- link algebra with generalised arithmetic eg for the commutative property, determine that $a + b = b + a$ *(Reflecting)*
- determine equivalence of algebraic expressions by substituting a given number for the letter *(Applying Strategies, Reasoning)*

Background Information

To gain an understanding of algebra, students must be introduced to the concepts of patterns, relationships, variables, expressions, unknowns, equations and graphs in a wide variety of contexts. For each successive context, these ideas need to be redeveloped. Students need gradual exposure to abstract ideas as they begin to relate algebraic terms to real situations.

It is important to develop an understanding of the use of letters as algebraic symbols for variable numbers of objects rather than for the objects themselves. The practice of using the first letter of the name of an object as a symbol for the number of such objects (or still worse as a symbol for the object) can lead to misconceptions and should be avoided, especially in the early Stages.

Introducing Letters as Algebraic Symbols

The recommended approach is to spend time over the conventions for using algebraic symbols for first-degree expressions and to situate the translation of generalisations from words to symbols as an application of students' knowledge of the symbol system rather than as an introduction to the symbol system.

Considerable time needs to be spent manipulating concrete materials, such as cups and counters, to develop a good understanding of the notion of a variable and to establish the equivalence of expressions.

The recommended steps for moving into symbolic algebra are:
- the variable notion, associating letters with a variety of variables
- symbolism for a variable plus a constant
- symbolism for a variable times a constant
- symbolism for sums and products.

When evaluating expressions, there must be an explicit direction to replace the letter by a number to ensure full understanding of notation occurs.

Thus if $a = 6, a + a = 6 + 6$ but $2a = 2\times 6$ and **not** 26.

It is suggested that the introduction of the symbol system precede the Number Patterns topic for Stage 4, since this topic presumes students are able to manipulate algebraic symbols and will use them to generalise patterns.

Number Patterns **Stage 4**

PAS4.2

Creates, records, analyses and generalises number patterns using words and algebraic symbols in a variety of ways

Key Ideas

Create, record and describe number patterns using words

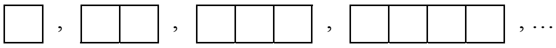
Use algebraic symbols to translate descriptions of number patterns

Represent number pattern relationships as points on a grid

Knowledge and Skills

Students learn about

- using a process that consists of building a geometric pattern, completing a table of values, describing the pattern in words and algebraic symbols and representing the relationship on a graph:
  - modelling geometric patterns using materials such as matchsticks to form squares

    eg 
  - describing the pattern in a variety of ways that relate to the different methods of building the squares, and recording descriptions using words
  - forming and completing a table of values for the geometric pattern

    eg

Number of squares	1	2	3	4	5	10	100
Number of matchsticks	4	7	10	13	_	_	_

  - representing the values from the table on a number grid and describing the pattern formed by the points on the graph (note – the points should not be joined to form a line because values between the points have no meaning)
  - determining a rule in words to describe the pattern from the table – this needs to be expressed in function form relating the top-row and bottom-row terms in the table
  - describing the rule in words, replacing the varying number by an algebraic symbol
  - using algebraic symbols to create an equation that describes the pattern
  - creating more than one equation to describe the pattern
  - using the rule to calculate the corresponding value for a larger number
- using a process that consists of identifying a number pattern (including decreasing patterns), completing a table of values, describing the pattern in words and algebraic symbols, and representing the relationship on a graph:
  - completing a table of values for the number pattern

    eg

a	1	2	3	4	5	10	100
b	4	7	10	13	_	_	_

Working Mathematically

Students learn to

- ask questions about how number patterns have been created and how they can be continued *(Questioning)*
- generate a variety of number patterns that increase or decrease and record them in more than one way *(Applying Strategies, Communicating)*
- model and then record number patterns using diagrams, words and algebraic symbols *(Communicating)*
- check pattern descriptions by substituting further values *(Reasoning)*
- describe the pattern formed by plotting points from a table and suggest another set of points that might form the same pattern *(Communicating, Reasoning)*
- describe what has been learnt from creating patterns, making connections with number facts and number properties *(Reflecting)*
- play ‘guess my rule’ games, describing the rule in words and algebraic symbols where appropriate *(Applying Strategies, Communicating)*
- represent and apply patterns and relationships in algebraic forms *(Applying Strategies, Communicating)*
- explain why a particular relationship or rule for a given pattern is better than another *(Reasoning, Communicating)*
- distinguish between graphs that represent an increasing number pattern and those that represent a decreasing number pattern *(Communicating)*
- determine whether a particular number pattern can be described using algebraic symbols *(Applying Strategies, Communicating)*

Number Patterns *(continued)*	**Stage 4**

- describing the pattern in a variety of ways and recording descriptions using words
- representing the values from the table on a number grid and describing the pattern formed by the points on the graph
- determining a rule in words to describe the pattern from the table – this needs to be expressed in function form relating the top-row and bottom-row terms in the table
- describing the rule in words, replacing the varying number by an algebraic symbol
- using algebraic symbols to create an equation that describes the pattern
- creating more than one equation to describe the pattern
- using the rule to calculate the corresponding value for a larger number

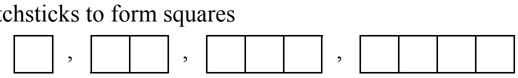
Background Information

It is recommended that students begin this topic with simple examples from PAS3.1a before attempting the more challenging patterns in this topic.

In completing tables, intermediate stages should be encouraged.

Consider the following example of the line of squares that is presented in the 'learn about' statements:

1 modelling geometric patterns using materials such as matchsticks to form squares

eg  , …

2 forming and completing a table of values for the geometric pattern

eg

Number of squares	1	2	3	4	5	10	100
Number of matchsticks	4	7	10	13	_	_	_

It may help students to develop the table as follows:

(i) starting from one match

Number of squares	1	2	3
Number of matches	$1 + 3 = 4$	$1 + 3 + 3 = 7$	$1 + 3 + 3 + 3 = 10$
or	$1 + 3 \times 1 = 4$	$1 + 3 \times 2 = 7$	$1 + 3 \times 3 = 10$
or	$1 + 1 \times 3 = 4$	$1 + 2 \times 3 = 7$	$1 + 3 \times 3 = 10$
or	…		

or:

(ii) starting from one square

Number of squares	1	2	3
Number of matches	4	$4 + 3 \times 1 = 7$	$4 + 3 \times 2 = 10$

Students recognise relationships in the table of values and extend the table to include cases that would be impractical to build, basing their calculations on their own verbal descriptions of the pattern eg for 102 squares, method (i) would lead to $1 + 3 \times 102 = 307$ and method (ii) would lead to $4 + 3 \times 101 = 307$.

Similarly, number patterns may be used as sources for verbal generalisations. Emphasis should be given to encouraging students to describe how they can obtain one term from earlier terms.

For example, in the number pattern 1, 3, 5, 7, 9, …'you keep adding two to get the next number'

1	$1 + 2$	$1 + 2 + 2$	$1 + 2 + 2 + 2$
or 1	$1 + 2 \times 1$	$1 + 2 \times 2$	$1 + 2 \times 3$
or 1	…		

Students could build the pattern using concrete materials or represent it using diagrams.

More than one aspect of a geometric pattern may be considered eg perimeter, area, number of corners.

The number plane is introduced in Linear Relationships (PAS4.5). Students could be introduced to the early ideas in that topic before graphing points in this topic.

Algebraic Techniques	**Stage 4**
PAS4.3 Uses the algebraic symbol system to simplify, expand and factorise simple algebraic expressions	**Key Ideas** Use the algebraic symbol system to simplify, expand and factorise simple algebraic expressions Substitute into algebraic expressions

Knowledge and Skills

Students learn about

- recognising like terms and adding and subtracting like terms to simplify algebraic expressions
  eg $2n+4m+n=4m+3n$
- recognising the role of grouping symbols and the different meanings of expressions, such as $2a+1$ and $2(a+1)$
- simplifying algebraic expressions that involve multiplication and division
  eg $12a \div 3$
  $4x \times 3$
  $2ab \times 3a$
- simplifying expressions that involve simple algebraic fractions
  eg $\frac{a}{2}+\frac{a}{3}$
  $\frac{2x}{5}-\frac{x}{3}$
- expanding algebraic expressions by removing grouping symbols (the distributive property)
  eg $3(a+2)=3a+6$
  $-5(x+2)=-5x-10$
  $a(a+b)=a^2+ab$
- factorising a single term eg $6ab=3\times 2\times a\times b$
- factorising algebraic expressions by finding a common factor
  eg $6a+12=6(a+2)$
  $x^2-5x=x(x-5)$
  $5ab+10a=5a(b+2)$
  $-4t-12=-4(t+3)$
- distinguishing between algebraic expressions where letters are used as variables, and equations, where letters are used as unknowns
- substituting into algebraic expressions
- generating a number pattern from an algebraic expression
  eg

x	1	2	3	4	5	6	10	100
$x+3$	4	5	6	_	_	_	_	_

- replacing written statements describing patterns with equations written in algebraic symbols
  eg 'you add five to the first number to get the second number' could be replaced with '$y=x+5$'
- translating from everyday language to algebraic language and from algebraic language to everyday language

Working Mathematically

Students learn to

- generate a variety of equivalent expressions that represent a particular situation or problem *(Applying Strategies)*
- determine and justify whether a simplified expression is correct by substituting numbers for letters *(Applying Strategies, Reasoning)*
- check expansions and factorisations by performing the reverse process *(Reasoning)*
- interpret statements involving algebraic symbols in other contexts eg creating and formatting spreadsheets *(Communicating)*
- explain why a particular algebraic expansion or factorisation is incorrect *(Reasoning, Communicating)*
- determine whether a particular pattern can be described using algebraic symbols *(Applying Strategies, Communicating)*

Algebraic Techniques **Stage 4**

PAS4.4

Uses algebraic techniques to solve linear equations and simple inequalities

Key Ideas

Solve linear equations and word problems using algebra
Solve simple inequalities

Knowledge and Skills

Students learn about

- solving simple linear equations using concrete materials, such as the balance model or cups and counters, stressing the notion of doing the same thing to both sides of an equation
- solving linear equations using strategies such as guess, check and improve, and backtracking (reverse flow charts)
- solving equations using algebraic methods that involve up to and including three steps in the solution process and have solutions that are not necessarily whole numbers

  eg
  $$3x + 4 = 13$$
  $$5(a + 3) = 14$$
  $$\frac{3t - 2}{5} = 6$$
  $$3g - 5 = g + 7$$
- checking solutions to equations by substituting
- translating a word problem into an equation, solving the equation and translating the solution into an answer to the problem
- solving equations arising from substitution into formulae

  eg given $P = 2l + 2b$ and $P = 20$, $l = 6$, solve for b
- finding a range of values that satisfy an inequality using strategies such as 'guess and check'
- solving simple inequalities such as
  $$6a \le 18$$
  $$5y < 14$$
  $$\frac{t}{5} \ge -2$$
- representing solutions to simple inequalities on the number line

Working Mathematically

Students learn to

- compare and contrast different methods to solve a range of linear equations *(Reasoning)*
- create equations to solve a variety of problems, clearly stating the meaning of introduced letters as 'the number of …', and verify solutions *(Applying Strategies, Reasoning)*
- use algebraic techniques as a tool for problem solving *(Applying Strategies)*
- construct formulae for finding areas of common geometric figures eg area of a triangle *(Applying Strategies)*
- determine equations that have a given solution eg find equations that have the solution *x = 5* *(Applying Strategies)*
- substitute into formulae used in other strands of the syllabus or in other key learning areas and interpret the solutions

  eg
  $$c^2 = a^2 + b^2$$
  $$S = \frac{D}{T}$$
  $$C = \frac{5}{9}(F - 32)$$

  (Applying Strategies, Communicating)
- describe the process of solving simple inequalities and justifying solutions *(Communicating, Reasoning)*

Background Information

Five models have been proposed to assist students with the solving of simple equations.

Model 1 uses a two-pan balance and objects such as coins or centicubes. A light paper wrapping can hide a 'mystery number' of objects without distorting the balance's message of equality.

Model 2 uses small objects (all the same) with some hidden in containers to produce the 'unknowns' or 'mystery numbers'.

eg place the same number of small objects in a number of paper cups and cover them with another cup. Form an equation using the cups and then remove objects in equal amounts from each side of a marked equals sign.

Model 3 uses one-to-one matching of terms on each side of the equation.

eg
$$3x + 1 = 2x + 3$$
$$x + x + x + 1 = x + x + 2 + 1$$
giving $x = 2$ through one-to-one matching.

Model 4 uses a substitution approach. By trial and error a value is found for the unknown that produces equality for the values of the two expressions on either side of the equation (this highlights the variable concept).

Simple equations can usually be solved using arithmetic methods. Students need to solve equations where the solutions are not whole numbers and that require the use of algebraic methods.

Model 5 uses backtracking or a reverse flow chart to unpack the operations and find the solution. This model only works for equations with all letters on the same side.

eg $3d + 5 = 17$

$17 - 5 \rightarrow \square \div 3 \rightarrow \square$

$17 - 5 \rightarrow 12 \div 3 \rightarrow 4$

$\therefore d = 4$

Algebraic Techniques	**Stage 5.1**
PAS5.1.1 Applies the index laws to simplify algebraic expressions	**Key Ideas** Apply the index laws to simplify algebraic expressions (positive integral indices only)

Knowledge and Skills

Students learn about

- using the index laws previously established for numbers to develop the index laws in algebraic form

eg

$2^2 \times 2^3 = 2^{2+3} = 2^5$	$a^m \times a^n = a^{m+n}$
$2^5 \div 2^2 = 2^{5-2} = 2^3$	$a^m \div a^n = a^{m-n}$
$\left(2^2\right)^3 = 2^6$	$(a^m)^n = a^{mn}$

- establishing that $a^0 = 1$ using the index laws

  eg $a^3 \div a^3 = a^{3-3} = a^0$

  and $a^3 \div a^3 = 1$

  $\therefore$ $a^0 = 1$

- simplifying algebraic expressions that include index notation

  eg $5x^0 + 3 = 8$

  $2x^2 \times 3x^3 = 6x^5$

  $12a^6 \div 3a^2 = 4a^4$

  $2m^3(m^2 + 3) = 2m^5 + 6m^3$

Working Mathematically

Students learn to

- verify the index laws using a calculator eg use a calculator to compare the values of $(3^4)^2$ and 3^8 *(Reasoning)*
- explain why $x^0 = 1$ *(Applying Strategies, Reasoning, Communicating)*
- link use of indices in Number with use of indices in Algebra *(Reflecting)*
- explain why a particular algebraic sentence is incorrect eg explain why $a^3 \times a^2 = a^6$ is incorrect *(Communicating, Reasoning)*
- examine and discuss the difference between expressions such as

  $3a^2 \times 5a$ and $3a^2 + 5a$

  by substituting values for a *(Reasoning, Applying Strategies, Communicating)*

Background Information

The index laws for numbers were established in the Number strand (NS4.1 and NS5.1.1).

These need to be used to assist the generalisations written using algebraic notation.

Algebraic Techniques	**Stage 5.2**
PAS5.2.1 Simplifies, expands and factorises algebraic expressions involving fractions and negative and fractional indices	**Key Ideas** Simplify, expand and factorise algebraic expressions including those involving fractions or with negative and/or fractional indices

Knowledge and Skills

Students learn about

- simplifying algebraic expressions involving fractions, such as

$$\frac{2x}{5}+\frac{2x}{3}$$
$$\frac{7a}{8}-\frac{5a}{12}$$
$$\frac{2y}{3}\div\frac{y}{6}$$
$$\frac{2ab}{3}\times\frac{6}{2b}$$

- applying the index laws to simplify expressions involving pronumerals
- establishing that

$$\left(\sqrt{a}\right)^2=\sqrt{a}\times\sqrt{a}=\sqrt{a\times a}=\sqrt{a^2}=a$$

- using index laws to assist with the definition of the fractional index for square root

given $\left(\sqrt{a}\right)^2=a$

and $\left(a^{\frac{1}{2}}\right)^2=a$

then $\sqrt{a}=a^{\frac{1}{2}}$

- using index laws to assist with the definition of the fractional index for cube root
- using index notation and the index laws to establish that

$$a^{-1}=\frac{1}{a},\ a^{-2}=\frac{1}{a^2},\ a^{-3}=\frac{1}{a^3},\ \ldots$$

- applying the index laws to simplify algebraic expressions such as

$$(3y^2)^3$$
$$4b^{-5}\times 8b^{-3}$$
$$9x^{-4}\div 3x^3$$
$$3x^{\frac{1}{2}}\times 5x^{\frac{1}{2}}$$
$$6y^{\frac{1}{3}}\div 4y^{\frac{1}{3}}$$

Working Mathematically

Students learn to

- describe relationships between the algebraic symbol system and number properties
  (Reflecting, Communicating)
- link algebra with generalised arithmetic
  eg use the distributive property of multiplication over addition to determine that

$$a(b+c)=ab+ac$$

  (Reflecting)
- determine and justify whether a simplified expression is correct by substituting numbers for pronumerals
  (Applying Strategies, Reasoning)
- generate a variety of equivalent expressions that represent a particular situation or problem
  (Applying Strategies)
- explain why finding the square root of an expression is the same as raising the expression to the power of a half
  (Communicating, Reasoning)
- state whether particular equivalences are true or false and give reasons
  eg Are the following true or false? Why?

$$5x^0=1$$
$$9x^5\div 3x^5=3x$$
$$a^5\div a^7=a^2$$
$$2c^{-4}=\frac{1}{2c^4}$$

  (Applying Strategies, Reasoning, Communicating)
- explain the difference between particular pairs of algebraic expressions, such as x^{-2} and $-2x$
  (Reasoning, Communicating)
- check expansions and factorisations by performing the reverse process *(Reasoning)*
- interpret statements involving algebraic symbols in other contexts eg spreadsheets *(Communicating)*
- explain why an algebraic expansion or factorisation is incorrect eg Why is the following incorrect?
  $24x^2y+16xy^2=8xy(3x-2)$
  (Reasoning, Communicating)

Algebraic Techniques ***(continued)***	**Stage 5.2**
• expanding, by removing grouping symbols, and collecting like terms where possible, algebraic expressions such as $2y(y-5)+4(y-5)$ $4x(3x+2)-(x-1)$ $-3x^2(5x^2+2xy)$ • factorising, by determining common factors, algebraic expressions such as $3x^2-6x$ $14ab+12a^2$ $21xy-3x+9x^2$	

Algebraic Techniques	**Stage 5.2**
PAS5.2.2 Solves linear and simple quadratic equations, solves linear inequalities and solves simultaneous equations using graphical and analytical methods	**Key Ideas** Solve linear and simple quadratic equations of the form $ax^2 = c$ Solve linear inequalities Solve simultaneous equations using graphical and analytical methods for simple examples

Knowledge and Skills

Students learn about

Linear and Quadratic Equations

- solving linear equations such as

$$\frac{x}{2}+\frac{x}{3}=5$$
$$\frac{2y-3}{3}=-2$$
$$\frac{z-3}{2}+6=1$$
$$3(a+2)+2(a-5)=10$$
$$3(2t-5)=2t+5$$
$$\frac{3r+1}{4}=\frac{2r-4}{5}$$

- solving word problems that result in equations
- exploring the number of solutions that satisfy simple quadratic equations of the form $x^2 = c$
- solving simple quadratic equations of the form $ax^2 = c$
- solving equations arising from substitution into formulae

Linear Inequalities

- solving inequalities such as

$$3x-1<9$$
$$2(a+4)\ge 24$$
$$\frac{t+4}{5}>-3$$

Working Mathematically

Students learn to

- compare and contrast different methods of solving linear equations and justify a choice for a particular case *(Applying Strategies, Reasoning)*
- use a number of strategies to solve unfamiliar problems, including:
  - using a table
  - drawing a diagram
  - looking for patterns
  - working backwards
  - simplifying the problem and
  - trial and error *(Applying Strategies, Communicating)*
- solve non-routine problems using algebraic methods *(Communicating, Applying Strategies)*
- explain why a particular value could not be a solution to an equation *(Applying Strategies, Communicating, Reasoning)*
- create equations to solve a variety of problems and check solutions *(Communicating, Applying Strategies, Reasoning)*
- write formulae for spreadsheets *(Applying Strategies, Communicating)*
- solve and interpret solutions to equations arising from substitution into formulae used in other strands of the syllabus and in other subjects. Formulae such as the following could be used:

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$E=\frac{1}{2}mv^2$$
$$V=\frac{4}{3}\pi r^3$$
$$SA=2\pi r^2+2\pi rh$$

(Applying Strategies, Communicating, Reflecting)

- explain why quadratic equations could be expected to have two solutions *(Communicating, Reasoning)*
- justify a range of solutions to an inequality *(Applying Strategies, Communicating, Reasoning)*

Algebraic Techniques *(continued)*	**Stage 5.2**
Simultaneous Equations • solving simultaneous equations using non-algebraic methods, such as 'guess and check', setting up tables of values or looking for patterns • solving linear simultaneous equations by finding the point of intersection of their graphs • solving simple linear simultaneous equations using an analytical method eg solve the following $3a + b = 17$ $2a - b = 8$ • generating simultaneous equations from simple word problems	• use graphics calculators and spreadsheet software to plot pairs of lines and read off the point of intersection *(Applying Strategies)* • solve linear simultaneous equations resulting from problems and interpret the results *(Applying Strategies, Communicating)*
Background Information Graphics calculators and computer graphing programs enable students to graph two linear equations and display the coordinates of the point of intersection.	

§ Algebraic Techniques	**Stage 5.3**
§ PAS5.3.1 Uses algebraic techniques to simplify expressions, expand binomial products and factorise quadratic expressions	**Key Ideas** Use algebraic techniques to simplify expressions, expand binomial products and factorise quadratic expressions

Knowledge and Skills

Students learn about

- simplifying algebraic expressions, including those involving fractions, such as

$$-11x+2y+7x-8y+5$$
$$4(3x+2)-(x-1)$$
$$4a-3b+\frac{2}{3}b-\frac{7a}{2}$$
$$\frac{x}{3}-\frac{x+1}{5}$$

- expanding binomial products by finding the area of rectangles

eg

	x	8
x	x^2	$8x$
3	$3x$	24

hence

$$(x+8)(x+3)=x^2+8x+3x+24$$
$$=x^2+11x+24$$

- using algebraic methods to expand a variety of binomial products, such as

$$(x+2)(x-3)$$
$$(2y+1)^2$$
$$(3a-1)(3a+1)$$

- recognising and applying the special products

$$(a+b)(a-b)=a^2-b^2$$
$$(a\pm b)^2=a^2\pm 2ab+b^2$$

- factorising expressions:
  - common factors
  - difference of two squares
  - perfect squares
  - trinomials
  - grouping in pairs for four-term expressions

Working Mathematically

Students learn to

- describe relationships between the algebraic symbol system and number properties *(Reflecting, Communicating)*
- develop facility with the algebraic symbol system in order to apply algebraic techniques to other strands and substrands *(Applying Strategies, Communicating)*
- link algebra with generalised arithmetic *(Reflecting)*
- determine and justify whether a simplified expression is correct by substituting numbers for pronumerals *(Applying Strategies, Reasoning)*
- generate a variety of equivalent expressions that represent a particular situation or problem *(Applying Strategies)*
- check expansions and factorisations by performing the reverse process *(Reasoning)*
- interpret statements involving algebraic symbols in other contexts eg spreadsheets *(Communicating)*
- use factorising techniques to solve quadratic equations and draw graphs of parabolas *(Applying Strategies, Communicating)*
- solve problems, such as:
  find a relationship that describes the number of diagonals in a polygon with n sides
  (Applying Strategies)

§ Algebraic Techniques *(continued)*	**Stage 5.3**
• using a variety of methods, including combinations of the above, to factorise expressions such as $3d^3 - 3d$ $2a^2 + 12a + 18$ $4x^2 - 20x + 25$ $6a^2 + 13a - 5$ $t^2 - 3t + st - 3s$ $2a^2b - 6ab - 3a + 9$ • factorising and simplifying a variety of more complex algebraic expressions such as $\frac{x^2 + 3x + 2}{x + 2}$ $\frac{4}{x^2 + x} - \frac{3}{x^2 - 1}$ $\frac{3m - 6}{4} \times \frac{8m}{m^2 - 2m}$ $\frac{4}{x^2 - 9} + \frac{2}{3x + 9}$	

§ Algebraic Techniques	Stage 5.3
§ PAS5.3.2 Solves linear, quadratic and simultaneous equations, solves and graphs inequalities, and rearranges literal equations	**Key Ideas** Solve quadratic equations by factorising, completing the square, or using the quadratic formula Solve a range of inequalities and rearrange literal equations Solve simultaneous equations including quadratic equations

Knowledge and Skills

Students learn about

Linear, Quadratic and Simultaneous Equations

- using analytical and graphical methods to solve a range of linear equations, including equations that involve brackets and fractions such as

$$3(2a-6)=5-(a+2)$$
$$\frac{2x-5}{3}-\frac{x+7}{5}=0$$
$$\frac{y-1}{4}-\frac{2y+3}{3}=\frac{1}{2}$$

- solving problems involving linear equations
- developing the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- solving equations of the form $ax^2+bx+c=0$ using:
  - factors
  - completing the square
  - the quadratic formula
- solving a variety of quadratic equations such as

$$3x^2=4$$
$$x^2-8x-4=0$$
$$x(x-4)=4$$
$$(y-2)^2=9$$

- identifying whether a given quadratic equation has no solution, one solution or two solutions
- checking the solutions of quadratic equations
- generating quadratic equations from problems
- solving problems involving quadratic equations
- solving quadratic equations resulting from substitution into formulae
- using analytical methods to solve a variety of simultaneous equations, including those that involve a first degree equation and a second degree equation, such as

$$3x-4y=2 \text{ and } 2x+y=3$$
$$y=x^2 \text{ and } y=x$$
$$y=x^2-x-2 \text{ and } y=x+6$$

Working Mathematically

Students learn to

- solve non-routine problems using algebraic techniques *(Applying Strategies, Communicating)*
- create equations to solve a variety of problems and check solutions *(Communicating, Applying Strategies, Reasoning)*
- explain why a particular value could not be a solution to an equation *(Applying Strategies, Communicating, Reasoning)*
- determine quadratic expressions to describe particular number patterns such as $y=x^2+1$ for the table

0	1	2	3	4	5
1	2	5	10	17	26

(Applying Strategies, Communicating)

- choose the most appropriate method to solve a particular quadratic equation *(Applying Strategies)*
- solve quadratic equations and discuss the possible number of roots for any quadratic equation *(Applying Strategies, Communicating)*
- graph simultaneous equations to find solutions and compare this method with analytic methods *(Applying Strategies, Reflecting, Communicating)*

§ Algebraic Techniques *(continued)* **Stage 5.3**

Inequalities

- using $<$, $>$, $\leq$, $\geq$, $\neq$ to generate linear inequalities from problems
- solving linear inequalities analytically, including changing the direction of the inequality when multiplying or dividing by a negative number in inequalities such as
  $1-4y\leq 6$
- solving problems involving inequalities (in one variable)

Literal Equations

- changing the subject of a formula, using examples from other strands and other subjects
  eg make r the subject of $\frac{1}{x}=\frac{1}{r}+\frac{1}{s}$,
  make b the subject of $x=\sqrt{b^2-4ac}$
- determining restrictions on the values of variables implicit in the original formula and after rearrangement of the formula
  eg consider what restrictions there would be on the variables in the equation $Z=ax^2$ and what additional restrictions are assumed if the equation is rearranged to
  $$x=\sqrt{\frac{Z}{a}}$$

Understanding Variables

- replacing variables with other expressions
  eg find an expression for x^2+4 if $x=2at$
- using variable substitution to simplify expressions and equations so that specific cases can be seen to belong to general categories eg substitute u for x^2
  to solve $x^4-13x^2+36=0$
- interpreting expressions and equations given additional information

- use a numerical example to justify the need to reverse the direction of the inequality when multiplying or dividing by a negative number *(Reasoning)*

Linear Relationships	**Stage 4**
PAS4.5 Graphs and interprets linear relationships on the number plane	**Key Ideas** Interpret the number plane and locate ordered pairs Graph and interpret linear relationships created from simple number patterns and equations
Knowledge and Skills **Students learn about** • interpreting the number plane formed from the intersection of a horizontal x -axis and vertical y -axis and recognising similarities and differences between points located in each of the four quadrants • identifying the point of intersection of the two axes as the origin, having coordinates (0,0) • reading, plotting and naming ordered pairs on the number plane including those with values that are not whole numbers • graphing points on the number plane from a table of values, using an appropriate scale • extending the line joining a set of points to show that there is an infinite number of ordered pairs that satisfy a given linear relationship • interpreting the meaning of the continuous line joining the points that satisfy a given number pattern • reading values from the graph of a linear relationship to demonstrate that there are many points on the line • deriving a rule for a set of points that has been graphed on a number plane by forming a table of values or otherwise • forming a table of values for a linear relationship by substituting a set of appropriate values for either of the letters and graphing the number pairs on the number plane eg given $y = 3x + 1$, forming a table of values using $x = 0$, 1 and 2 and then graphing the number pairs on a number plane with appropriate scale • graphing more than one line on the same set of axes and comparing the graphs to determine similarities and differences eg parallel, passing through the same point • graphing two intersecting lines on the same set of axes and reading off the point of intersection	**Working Mathematically** **Students learn to** • relate the location of points on a number plane to maps, plans, street directories and theatre seating and note the different recording conventions eg 15°E *(Communicating, Reflecting)* • compare similarities and differences between sets of linear relationships *(Reasoning)* eg $y = 3x$, $y = 3x + 2$, $y = 3x - 2$ $y = x$, $y = 2x$, $y = 3x$ $y = -x$, $y = x$ • sort and classify equations of linear relationships into groups to demonstrate similarities and differences *(Reasoning)* • question whether a particular equation will have a similar graph to another equation and graph the line to check *(Questioning, Applying Strategies, Reasoning)* • recognise and explain that not all patterns form a linear relationship *(Reasoning)* • determine and explain differences between equations that represent linear relationships and those that represent non-linear relationships (*Applying Strategies, Reasoning*) • explain the significance of the point of intersection of two lines in relation to it being a solution of each equation *(Applying Strategies, Reasoning)* • question if the graphs of all linear relationships that have a negative x term will decrease *(Questioning)* • reason and explain which term affects the slope of a graph, making it either increasing or decreasing *(Reasoning, Communicating)* • use a graphics calculator and spreadsheet software to graph and compare a range of linear relationships *(Applying Strategies, Communicating)*
Background Information In this topic, the notion of locating position that was established in Stage 3 in the Space and Geometry strand is further developed to include negative numbers and the use of the four-quadrant number plane. While alternative grid systems may be used in early experiences, it is intended that the standard rectangular grid system be established.	In this topic, linear refers to straight lines. Investigate the use of coordinates by Descartes and Fermat to identify points in terms of positive or zero distances from axes. Isaac Newton introduced negative values.
Language Students will need to become familiar with and be able to use new terms including coefficient, constant term, and intercept.	

Coordinate Geometry	Stage 5.1
PAS5.1.2 Determines the midpoint, length and gradient of an interval joining two points on the number plane and graphs linear and simple non-linear relationships from equations	**Key Ideas** Use a diagram to determine midpoint, length and gradient of an interval joining two points on the number plane Graph linear and simple non-linear relationships from equations

Knowledge and Skills

Students learn about

Midpoint, Length and Gradient

- determining the midpoint of an interval from a diagram
- graphing two points to form an interval on the number plane and forming a right-angled triangle by drawing a vertical side from the higher point and a horizontal side from the lower point
- using the right-angled triangle drawn between two points on the number plane and Pythagoras' theorem to determine the length of the interval joining the two points
- using the right-angled triangle drawn between two points on the number plane and the relationship
$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$
to find the gradient of the interval joining two points
- determining whether a line has a positive or negative slope by following the line from left to right – if the line goes up it has a positive slope and if it goes down it has a negative slope
- finding the gradient of a straight line from the graph by drawing a right-angled triangle after joining two points on the line

Graphs of Relationships

- constructing tables of values and using coordinates to graph vertical and horizontal lines such as
$$x = 3,\ x = -1$$
$$y = 2,\ y = -3$$
- identifying the x - and y -intercepts of graphs
- identifying the x -axis as the line $y = 0$
- identifying the y -axis as the line $x = 0$
- graphing a variety of linear relationships on the number plane by constructing a table of values and plotting coordinates using an appropriate scale eg graph the following:
$$y = 3 - x$$
$$y = \frac{x+1}{2}$$
$$x + y = 5$$
$$x - y = 2$$
$$y = \frac{2}{3}x$$

Working Mathematically

Students learn to

- describe the meaning of the midpoint of an interval and how it can be found *(Communicating)*
- describe how the length of an interval joining two points can be calculated using Pythagoras' theorem *(Communicating, Reasoning)*
- explain the meaning of gradient and how it can be found for a line joining two points *(Communicating, Applying Strategies)*
- distinguish between positive and negative gradients from a graph *(Communicating)*
- relate the concept of gradient to the tangent ratio in trigonometry for lines with positive gradients *(Reflecting)*
- describe horizontal and vertical lines in general terms *(Communicating)*
- explain why the x -axis has equation $y = 0$ *(Reasoning, Communicating)*
- explain why the y -axis has equation $x = 0$ *(Reasoning, Communicating)*
- determine the difference between equations of lines that have a negative gradient and those that have a positive gradient *(Reasoning)*
- use a graphics calculator and spreadsheet software to graph, compare and describe a range of linear and simple non-linear relationships *(Applying Strategies, Communicating)*
- apply ethical considerations when using hardware and software *(Reflecting)*

Coordinate Geometry *(continued)*	**Stage 5.1**
• graphing simple non-linear relationships eg $y = x^2$ $y = x^2 + 2$ $y = 2^x$ • determining whether a point lies on a line by substituting into the equation of the line	

Background Information

The process of drawing vertical and horizontal lines to find gradients is applied in a variety of other situations eg finding angles of elevation or depression, gradients in calculus in later Stages, and plotting courses using compass bearings.

Students will have met travel, step and conversion graphs in the Data strand in Stage 4.

Midpoint links with average in Number.

Exponential graphs link with work on indices.

In Geography, students find the gradient between various locations using contour lines and the map scale.

Coordinate Geometry	**Stage 5.2**
PAS5.2.3 Uses formulae to find midpoint, distance and gradient and applies the gradient/intercept form to interpret and graph straight lines	**Key Ideas** Use midpoint, distance and gradient formulae Apply the gradient/intercept form to interpret and graph straight lines

Knowledge and Skills

Students learn about

Midpoint, Distance and Gradient Formulae

- using the average concept to establish the formula for the midpoint, *M*, of the interval joining two points (x_1, y_1) and (x_2, y_2) on the number plane

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- using the formula to find the midpoint of the interval joining two points on the number plane
- using Pythagoras' theorem to establish the formula for the distance, *d*, between two points (x_1, y_1) and (x_2, y_2) on the number plane

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- using the formula to find the distance between two points on the number plane
- using the relationship

$$\text{gradient} = \frac{\text{rise}}{\text{run}}$$

  to establish the formula for the gradient, *m*, of an interval joining two points (x_1, y_1) and (x_2, y_2) on the number plane

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- using the formula to find the gradient of an interval joining two points on the number plane

Working Mathematically

Students learn to

- explain the meaning of each of the pronumerals in the formulae for midpoint, distance and gradient *(Communicating)*
- use the appropriate formulae to solve problems on the number plane *(Applying Strategies)*
- use gradient and distance formulae to determine the type of triangle three points will form or the type of quadrilateral four points will form and justify the answer *(Applying Strategies, Reasoning)*
- explain why the following formulae give the same solutions as those in the left-hand column

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \text{ and}$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

(Reasoning, Communicating)

Coordinate Geometry *(continued)*	**Stage 5.2**
Gradient/Intercept Form • constructing tables of values and using coordinates to graph straight lines of the form $y = mx + b$ ie gradient/intercept form • recognising equations of the form $y = mx + b$ as representing straight lines and interpreting the x -coefficient *(m)* as the gradient and the constant *(b)* as the y -intercept • rearranging an equation in general form ($ax + by + c = 0$) to the gradient/intercept form • graphing equations of the form $y = mx + b$ using the y -intercept *(b)* and the gradient *(m)* • determining that two lines are parallel if their gradients are equal • finding the gradient and the y -intercept of a straight line from the graph and using them to determine the equation of the line	• determine the difference in equations of lines that have a negative gradient and those that have a positive gradient *(Reasoning)* • match equations of straight lines to graphs of straight lines and justify choices *(Communicating, Reasoning)* • compare similarities and differences between sets of linear relationships eg $y = -3x, \quad y = -3x + 2, \quad y = -3x - 2$ $y = \frac{1}{2}x, \quad y = -2x, \quad y = 3x$ $x = 2, \quad y = 2$ *(Reasoning)* • sort and classify a mixed set of equations of linear relationships into groups to demonstrate similarities and differences *(Reasoning, Communicating)* • conjecture whether a particular equation will have a similar graph to another equation and graph both lines to test the conjecture *(Questioning, Applying Strategies, Reasoning)* • explain the effect on the graph of a line of changing the gradient or y -intercept *(Reasoning, Communicating)* • use a graphics calculator and spreadsheet software to graph a variety of equations of straight lines, and compare and describe the similarities and differences between the lines *(Applying Strategies, Communicating, Reasoning)* • apply knowledge and skills of linear relationships to practical problems *(Applying Strategies)* • apply ethical considerations when using hardware and software *(Reflecting)*
Background Information The Cartesian plane is named after Descartes who was one of the first to develop analytical geometry on the number plane. He shared this honour with Fermat. Descartes and Fermat are recognised as the first modern mathematicians.	

Coordinate Geometry	**Stage 5.2**
PAS5.2.4 Draws and interprets graphs including simple parabolas and hyperbolas	**Key Ideas** Draw and interpret graphs including simple parabolas and hyperbolas
Knowledge and Skills **Students learn about** • generating simple quadratic relationships, compiling tables of values and graphing equations of the form $y = ax^2$ and $y = ax^2 + c$ • generating simple hyperbolic relationships, compiling tables of values and graphing equations of the form $y = \frac{k}{x}$ for integral values of k • identifying graphs of straight lines, parabolas and hyperbolas • matching graphs of straight lines, parabolas and hyperbolas to the appropriate equations	**Working Mathematically** **Students learn to** • identify parabolic shapes in the environment *(Reflecting)* • describe the effect on the graph of $y = x^2$ of multiplying by different constants or of adding different constants *(Reasoning, Communicating)* • discuss and predict the equation of a parabola from its graph, with the main features clearly marked, using computer graphing software *(Communicating)* • describe the effect on the graph of $y = \frac{1}{x}$ of multiplying by different constants *(Reasoning, Communicating)* • explain what happens to the y-values of the points on the hyperbola $y = \frac{k}{x}$ as the x-values get very large *(Reasoning, Communicating)* • explain what happens to the y-values of the points on the hyperbola $y = \frac{k}{x}$ as the x-values get closer to zero *(Reasoning, Communicating)* • sort and classify a set of graphs, match each graph to an equation, and justify each choice *(Reasoning, Communicating)* • explain why it may be useful to include small and large numbers when constructing a table of values eg 'For $y = \frac{1}{x}$, why do we need to use more than the integers 1, 2, 3, and 4 for x?' *(Reasoning, Communicating)* • use a graphics calculator and spreadsheet software to graph, compare and describe a range of linear and non-linear relationships *(Applying Strategies, Communicating)*

Background Information

Graphics calculators and various computer programs facilitate the investigation of the shapes of curves and the effect on the equation of multiplying by, or adding, a constant.

This topic could provide opportunities for modelling.

For example, the hyperbola $y = \frac{1}{x}$ for $x > 0$, models sharing a prize between x people, or length of a rectangle given area k and breadth x.

§ Coordinate Geometry	**Stage 5.3**
§ PAS5.3.3 Uses various standard forms of the equation of a straight line and graphs regions on the number plane	**Key Ideas** Use and apply various standard forms of the equation of a straight line, and graph regions on the number plane

Knowledge and Skills

Students learn about

Equations of a Straight Line

- describing the equation of a line as the relationship between the x - and y -coordinates of any point on the line
- finding the equation of a line passing through a point (x_1, y_1), with a given gradient *m*, using:

$$y - y_1 = m(x - x_1)$$
$$y = mx + b$$

- finding the equation of a line passing through two points
- recognising and finding the equation of a line in the general form: $ax + by + c = 0$
- rearranging equations from the general form to the gradient/intercept form and hence graphing the line
- rearranging equations in the gradient-intercept form to the general form
- sketching the graph of a line by finding the x - and y -intercepts from its equation
- demonstrating that two lines are perpendicular if the product of their gradients is –1
- finding the equation of a line that is parallel or perpendicular to a given line

Regions

- graphing inequalities of the form $y < a$, $y > a$, $y \le a$, $y \ge a$, $x < a$, $x > a$, $x \le a$ and $x \ge a$ on the number plane
- graphing inequalities such as $y \le x$ on the number plane by considering the position of the boundary of the region as the limiting case
- checking whether a particular point lies in a given region specified by a linear inequality
- graphing regions such as that specified by

$$x + y < 7$$
$$2x - 3y \ge 5$$

Working Mathematically

Students learn to

- recognise from a list of equations those that result in straight line graphs *(Communicating)*
- describe conditions for lines to be parallel or perpendicular *(Reasoning, Communicating)*
- show that if two lines are perpendicular then the product of their gradients is –1
  (Applying Strategies, Reasoning, Communicating)
- discuss the equations of graphs that can be mapped onto each other by a translation or by reflection in the y -axis eg consider the graphs

  $$y = 2x,\ y = -2x,\ y = 2x + 1$$

  and describe the transformation that would map one graph onto the other *(Communicating)*
- describe the conditions for a line to have a negative gradient *(Reasoning, Communicating)*
- prove that a particular triangle drawn on the number plane is right-angled *(Applying Strategies, Reasoning)*
- use a graphics calculator and spreadsheet software to graph, compare and describe a range of linear relationships *(Applying Strategies, Communicating)*
- apply ethical considerations when using hardware and software *(Reflecting)*

- find areas of shapes enclosed within a set of lines on the number plane eg find the area of the triangle enclosed by the lines $y = 0$, $y = 2x$, $x + y = 6$
  (Applying Strategies)
- describe a region from a graph by identifying the boundary lines and determining the appropriate inequalities for describing the enclosed region
  (Applying Strategies, Communicating)

§ Coordinate Geometry	**Stage 5.3**
§ PAS5.3.4 Draws and interprets a variety of graphs including parabolas, cubics, exponentials and circles and applies coordinate geometry techniques to solve problems	**Key Ideas** Draw and interpret a variety of graphs including parabolas, cubics, exponentials and circles Solve coordinate geometry problems

Knowledge and Skills

Students learn about

Graphs

- identifying a variety of graphs from their equations
- finding x - and y -intercepts for the graph of $y = ax^2 + bx + c$ given a, b and c
- graphing a range of parabolas, including where the equation is given in the form

$$y = ax^2 + bx + c$$

  for various values of a, b and c
- finding the equation of the axis of symmetry of a parabola by:
  - finding the midpoint of the interval joining the points at which the parabola cuts the x -axis
  - using the formula

$$x = -\frac{b}{2a}$$

- finding the coordinates of the vertex of a parabola by:
  - finding the midpoint of the interval joining the points at which the parabola cuts the x -axis and substituting
  - completing the square
  - using the formula for the axis of symmetry to obtain the x -coordinate and substituting to obtain the y -coordinate
- identifying and using features of parabolas and their equations to assist in sketching quadratic relationships eg x - and y -intercepts, vertex, axis of symmetry and concavity
- graphing equations of the form $y = ax^3 + d$ and describing the effect on the graph of different values of a and d
- sketching, comparing and describing the key features of simple exponential curves such as

$$y = 2^x$$
$$y = -2^x$$
$$y = 2^{-x}$$
$$y = -2^{-x}$$

- recognising and describing the algebraic equations that represent circles with centre the origin and radius r
- using Pythagoras' theorem to establish the equation of a circle, centre the origin, radius r and graph equations of the form $x^2 + y^2 = r^2$

Working Mathematically

Students learn to

- describe the graph of a parabola from its equation *(Communicating)*
- investigate and describe similarities and differences between the graphs of a variety of parabolas such as

$$y = x^2$$
$$y = x^2 \pm 2$$
$$y = (x-2)^2$$
$$y = (x+2)(x-2)$$

  (Questioning, Applying Strategies, Reasoning, Communicating)
- investigate the graphs of parabolas of the following forms to determine features

$$y = ax^2$$
$$y = ax^2 + k$$
$$y = (x \pm a)^2$$
$$y = (x \pm a)^2 + k$$

  (Applying Strategies)
- sort and classify a set of equations, match each equation to a graph, and justify each choice *(Applying Strategies, Reasoning)*
- discuss and predict a possible equation from a given graph and check using technology *(Applying Strategies, Reasoning)*
- draw and compare graphs using a graphics calculator and/or a computer graphing package *(Applying Strategies)*
- compare and contrast a mixed set of graphs and determine possible equations from key features

  eg

$$y = 2$$
$$y = 2 - x$$
$$y = 2x^2$$
$$y = (x-2)^2$$
$$y = x^3 - 2$$
$$y = 2^x$$
$$x^2 + y^2 = 4$$

  (Applying Strategies, Reasoning, Communicating)
- determine whether a particular point is inside, on, or outside a circle *(Applying Strategies, Reasoning)*

§ Coordinate Geometry *(continued)*	**Stage 5.3**
Coordinate Geometry Problems • solving a variety of problems by applying coordinate geometry formulae and reasoning	• derive the formula for the distance between two points *(Applying Strategies, Reasoning)* • show that two intervals with equal gradients and a common point form a straight line and use this to show that three points are collinear *(Applying Strategies, Reasoning)* • use coordinate geometry to investigate and describe the properties of triangles and quadrilaterals *(Applying Strategies, Reasoning, Communicating)* • use coordinate geometry to investigate the intersection of the perpendicular bisectors of the sides of acute-angled triangles *(Applying Strategies, Reasoning, Communicating)* • show that four specified points form the vertices of particular quadrilaterals *(Applying Strategies, Reasoning, Communicating)*
Background Information Links to other key learning areas and real life examples of graphs eg exponential graphs used for population growth in demographics, radioactive decay, town planning, etc.	This topic could provide opportunities for modelling. For example, $y = 1.2^x$ for $x \geq 0$, models the growth of a quantity beginning at 1 and increasing 20% for each unit increase in x.

Graphs of Physical Phenomena	**Stage 5.2**
PAS5.2.5 Draws and interprets graphs of physical phenomena	**Key Ideas** Draw and interpret graphs of physical phenomena

Knowledge and Skills	**Working Mathematically**
Students learn about • interpreting distance/time graphs made up of straight line segments • determining which variable should be placed on the horizontal axis • drawing distance/time graphs • telling a story shown by a graph by describing how one quantity varies with the other eg number of cars at a red light, the temperature of water in a storage heater • sketching informal graphs to model familiar events eg noise level within the classroom during the lesson • using the relative positions of two points on a graph, rather than a detailed scale, to interpret information	**Students learn to** • describe the meaning of different gradients for the graph of a particular event *(Communicating)* • distinguish between positive and negative gradients from a graph *(Communicating)* • match a graph to a description of a particular event and explain reasons for the choice (*Reasoning, Communicating)* • compare graphs of the same simple situation, decide which one is the most appropriate and explain why *(Applying Strategies, Reasoning, Communicating)* • use spreadsheets to generate examples of everyday graphs *(Applying Strategies)* • model, record data and sketch graphs to investigate the distance of a moving object from a fixed point in relation to time eg move along a measuring tape for 30 seconds using a variety of activities that involve a constant rate such as: - walking slowly - walking for 10 seconds, stopping for 10 seconds and continuing at the same rate for the remaining 10 seconds to the end of the tape - walking for 10 seconds, stopping for 10 seconds and then turning around and walking back to the beginning of the tape for 10 seconds - starting at the other end of the line and walking back towards the beginning at a constant speed and record the distance at fixed time intervals so that a graph can be drawn to represent each situation *(Applying Strategies, Communicating)* • use technology such as data loggers to collect data for constant speeds and graph the data to compare and contrast graphs *(Applying Strategies, Reasoning)*

Background Information

Links with work done in the Data strand.

At this Stage, the focus is on examining situations where the data yields a constant rate of change. It is possible that some practical situations may yield a variable rate of change. This is the focus at the next Stage in PAS5.3.5.

Data loggers are used in Science for the collection of data and should be readily available in schools.

It is the usual practice in mathematics to place the independent variable on the horizontal axis and the dependent variable on the vertical axis. This is not always the case in other subjects eg Economics.

Graphs of Physical Phenomena	Stage 5.3
PAS5.3.5 Analyses and describes graphs of physical phenomena	**Key Ideas** Analyse and describe graphs of physical phenomena

Knowledge and Skills

Students learn about

- interpreting distance/time graphs when the speed is variable
- analysing the relationship between variables as they change over time
  eg draw graphs to represent the relationship between the depth of water in containers of different shapes when they are filled at a constant rate
- interpreting graphs, making sensible statements about the rate of increase or decrease, the initial and final points, constant relationships as denoted by straight lines, variable relationships as denoted by curved lines, etc
- describing qualitatively the rate of change of a graph using terms such as ‘increasing at a decreasing rate’
- sketching a graph from a simple description given a variable rate of change

Working Mathematically

Students learn to

- decide whether a particular graph is a suitable representation of a given physical phenomenon *(Communicating)*
- match a set of distance/time graphs to a set of descriptions and give reasons for choices *(Applying Strategies, Reasoning, Reflecting, Communicating)*
- model, record data and sketch graphs to investigate the distance of a moving object from a fixed point in relation to time
  eg move along a measuring tape for 30 seconds in a variety of activities (including variable speeds) such as:
  - running as fast as possible
  - walking slowly
  - walking for 10 seconds, stopping for 10 seconds and walking for the remaining 10 seconds
  - starting at the other end of the line and walking back towards the beginning
  - walking slowly for 10 seconds and then speeding up until the end of the tape
  - running at a decreasing speed

  and record the distance at fixed time intervals so that a graph can be drawn to represent each situation
  (Applying Strategies, Communicating)
- match a set of distance/time graphs to situations, as in the example above, and discuss the likelihood that they are accurate, appropriate, and whether they are possible
  (Applying Strategies, Communicating, Reasoning)
- use technology such as data loggers to collect data for variable speeds and graph the data to compare and contrast the graphs
  (Applying Strategies, Reasoning)

Background Information

Rate of change can be linked to rates met in the Number strand. In this section, rate of change is considered as it occurs in practical situations, including population growth and travel. Simple linear models have a constant rate of change. In other situations, the rate of change is variable.

Data loggers are used in Science for the collection of data and should be readily available in schools.

This topic is intended to provide experiences for students that will give them an intuitive understanding of rates of change and will assist the development of appropriate vocabulary. No quantitative analysis is needed at this Stage.

# Curve Sketching and Polynomials	Stage 5.3
# PAS5.3.6 Uses a variety of techniques to sketch a range of curves and describes the features of curves from the equation	**Key Ideas** Sketch a range of polynomials
Knowledge and Skills	**Working Mathematically**
Students learn about • graphing, and comparing the features of the graphs of equations of the form $y = ax^3$ $y = ax^3 + d$ $y = a(x-r)(x-s)(x-t)$ • relating the roots of the equation to the x-intercepts and relating the constant term to the y-intercept • graphing a variety of equations of the form $y = ax^n$ for $n > 0$, describing the effect of n being odd or even on the shape of the curve • graphing curves of the form $y = ax^n + k$ from curves of the form $y = ax^n$ by vertical transformations • graphing curves of the form $y = a(x - r)^n$ after first graphing curves of the form $y = ax^n$ and then using horizontal transformations • establishing the equation of the circle centre (h,k), radius r, and graphing equations of the form $(x-h)^2 + (y-k)^2 = r^2$ • recognising and describing equations that represent circles • finding the centre and radius of a circle whose equation is in the form $x^2 + gx + y^2 + hy = c$, by completing the square • determining the points of intersection of a line with a parabola, circle or hyperbola, graphically and algebraically • describing graphs in terms of what happens to the y-values as x becomes very large or very small, whether there is a y-value for every x-value, and what occurs near or at $x = 0$	**Students learn to** • describe how to sketch a particular curve by determining its features from the equation *(Applying Strategies, Communicating)* • describe the effect on a particular curve of multiplying x by a value a, or of adding a constant d *(Reasoning, Communicating)* • describe the shape and features of a curve in words from the equation *(Applying Strategies, Reasoning, Communicating)* • compare, describe and sketch a mixed set of equations such as $y = -3x$ $y = 3x^3$ $y = (x-3)^3$ $y = \frac{3}{x}$ $y = 3^x$ $x^2 + y^2 = 3$ $(x-3)^2 + (y-3)^2 = 3$ *(Applying Strategies, Reasoning, Communicating)* • identify equations that have a graph that is symmetrical about the y-axis *(Applying Strategies)* • compare methods of finding points of intersection of curves and justify choice of method for a particular example *(Reasoning, Communicating)* • use a graphics calculator and spreadsheet software to graph, compare and describe a range of polynomials *(Applying Strategies, Communicating)*

# Curve Sketching and Polynomials	Stage 5.3
# PAS5.3.7 Recognises, describes and sketches polynomials, and applies the factor and remainder theorems to solve problems	**Key Ideas** Add, subtract, multiply and divide polynomials Apply the factor and remainder theorems

Knowledge and Skills

Students learn about

- recognising a polynomial expression
  $$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$
  and using the terms degree, leading term, leading coefficient, constant term and monic polynomial appropriately
- using the notation *P(x)* for polynomials and *P(c)* to indicate the value of *P(x)* for $x = c$
- adding and subtracting polynomials and multiplying polynomials by linear expressions
- dividing polynomials by linear expressions to find the quotient and remainder, expressing the polynomial as the product of the linear expression and another polynomial plus a remainder
  eg $P(x) = (x-a)\,Q\,(x) + c$
- verifying the remainder theorem and using it to find factors of polynomials
- using the factor theorem to factorise certain polynomials completely
  ie if $(x-a)$ is a factor of *P(x),* then *P(a)* = 0
- using the factor theorem and long division to find all zeros of a simple polynomial and hence solve $P(x) = 0$ (degree ≤ 4)
- stating the number of zeros that a polynomial of degree *n* can have
- sketching the graphs of quadratic, cubic and quartic polynomials by factorising and finding the zeros
- determining the effect of single, double and triple roots of a polynomial equation on the shape of the curve
- using the leading term, the roots of the equation and the x - and y -intercepts to sketch polynomials
- using the sketch of $y = P(x)$ to sketch
  $$y = -P(x)$$
  $$y = P(-x)$$
  $$y = P(x) + c$$
  $$y = aP(x)$$

Working Mathematically

Students learn to

- recognise linear, quadratic and cubic expressions as examples of polynomials and relate sketching of these curves to factorising polynomials and finding the zeros
  (Applying Strategies, Reasoning, Reflecting, Communicating)
- determine the importance of the sign of the leading term of the polynomial on the behaviour of the curve as $x \to \pm\infty$
  (Applying Strategies, Reasoning, Communicating)
- use a graphics calculator or software package to sketch polynomials of odd and even degree and investigate the relationship between the number of zeros and the degree of the polynomial
  (Applying Strategies, Communicating)
- describe the key features of a polynomial so the graph of the polynomial can be drawn from the description
  (Applying Strategies, Communicating)
- discuss the similarities and differences between the graphs of two polynomials such as
  $y = x^3 + x^2 + x,\ y = x^3 + x^2 + x + 1$
  (Applying Strategies, Reasoning, Communicating)

# Functions and Logarithms	Stage 5.3
# PAS5.3.8 Describes, interprets and sketches functions and uses the definition of a logarithm to establish and apply the laws of logarithms	**Key Ideas** Define functions Use function notation Determine inverse functions Establish and apply the laws of logarithms

Knowledge and Skills

Students learn about

Functions

- defining a function as a rule or relationship where for each input there is only one output, or that associates every member of one set with exactly one member of the second set
- using the vertical line test on a graph to decide whether it represents a function
- using the notation *f(x)*
- using *f(c)* notation to determine the value of *f(x)* when $x = c$
- finding the permissible x - and y -values for a variety of functions (including functions represented by straight lines, parabolas, exponentials and hyperbolas)
- determining the inverse functions for a variety of functions and recognising their graphs as reflections in the line $y = x$
- sketching the graph of $y = f(x) + k$ and $y = f(x - a)$ given the graph of $y = f(x)$

Logarithms

The logarithm of a number to any positive base is the index when the number is expressed as a power of the base. Alternatively, $a^x = y \Leftrightarrow \log_a y = x$ *where* $a > 0, y > 0$.

- defining logarithms as indices and translating index statements into equivalent statements using logarithms eg

$$9 = 3^2, \quad \therefore \log_3 9 = 2$$
$$\frac{1}{2} = 2^{-1}, \quad \therefore \log_2 \frac{1}{2} = -1$$
$$4^{\frac{3}{2}} = 8, \quad \therefore \log_4 8 = \frac{3}{2}$$

- deducing the following laws of logarithms from the laws of indices:

$$\log_a x + \log_a y = \log_a (xy)$$
$$\log_a x - \log_a y = \log_a (\frac{x}{y})$$
$$\log_a x^n = n \log_a x$$

Working Mathematically

Students learn to

- sketch graphs to model relationships that occur in practical situations and explain the relationship between the variables represented in the graph *(Applying Strategies, Reasoning, Communicating)*
- consider a graph that represents a practical situation and explain the relationship between the two variables *(Applying Strategies, Reasoning, Communicating)*
- explain whether a given graph represents a function *(Applying Strategies, Reasoning, Communicating)*
- decide whether a straight line graph always, sometimes or never represents a function *(Applying Strategies, Communicating)*
- explain the conditions for a function to have an inverse function *(Applying Strategies, Reasoning, Communicating)*
- recognise and describe the restrictions that need to be placed on quadratic functions so that they have an inverse function *(Applying Strategies, Reasoning, Reflecting, Communicating)*

- relate logarithms to practical scales that use indices eg Richter, decibel and pH *(Applying Strategies, Communicating)*
- compare and contrast a set of exponential and logarithmic graphs drawn on the same axes to determine similarities and differences

  eg $y = 2^x$
  $y = \log_2 x$
  $y = 3^x$
  $y = \log_3 x$

  (Applying Strategies, Reasoning, Communicating)

# Functions and Logarithms *(continued)*	Stage 5.3
• establishing and using the following results: $\log_a a^x = x$ $\log_a a = 1$ $\log_a 1 = 0$ $\log_a (\frac{1}{x}) = -\log_a x$ • applying the laws of logarithms to evaluate simple expressions eg evaluate the following: $\log_2 8$ $\log_{81} 3$ $\log_{10} 25 + \log_{10} 4$ $3\log_{10} 2 + \log_{10}(12.5)$ $\log_2 18 - 2\log_2 3$ • simplifying expressions using the laws of logarithms eg simplify $5\log_a a - \log_a a^4$ • drawing the graphs of the inverse functions $y = a^x$ and $y = \log_a x$ • solving simple equations that contain exponents or logarithms eg $2^t = 8$, $4^{t+1} = \frac{1}{8\sqrt{2}}$ $\log_{27} 3 = x$, $\log_4 x = -2$	

Background Information

Teachers could consider the history and applications of logarithms.

Logarithm tables were used to assist with calculations before the use of hand-held calculators. They converted multiplication and division to addition and subtraction thus simplifying the calculations.

Language

Teachers need to emphasise the correct language used in connection with logarithms
eg $\log_a a^x = x$ is 'log to the base *a* of *a* to the power of *x* equals *x*'.

9.4 Data

In our contemporary society, there is a constant need for all people to understand, interpret and analyse information displayed in tabular or graphical forms. Students need to recognise how information may be displayed in a misleading manner resulting in false conclusions.

The Data strand extends from Early Stage 1 to Stage 5.2 and includes the collection, organisation, display and analysis of data. Early experiences are based on real-life contexts using concrete materials. This leads to data collection methods and the display of data in a variety of ways. Students are encouraged to ask questions relevant to their experiences and interests and to design ways of investigating their questions. Students should be aware of the extensive use of statistics in society. Print and Internet materials are useful sources of data that can be analysed and evaluated. Tools such as spreadsheets and other software packages may be used where appropriate to organise, display and analyse data.

This strand links to the topic Probability in the interpretation of the relative frequency of an event.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Stages 2 and 3 in one substrand. The Stage 4 content is presented in the topics: Data Representation and Data Analysis and Evaluation. The content for Stage 5.1 is represented in the topic Data Representation and Analysis while Stage 5.2 is represented in the topic Data Analysis and Evaluation.

Summary of Data Outcomes for Stages 2 to 5 with page references

Data

DS2.1 Gathers and organises data, displays data using tables and graphs, and interprets the results (p 112)

DS3.1 Displays and interprets data in graphs with scales of many-to-one correspondence (p 113)

Data Representation

DS4.1 Constructs, reads and interprets graphs, tables, charts and statistical information (p 114)

Data Analysis and Evaluation

DS4.2 Collects statistical data using either a census or a sample and analyses data using measures of location and range (p 115)

Data Representation and Analysis

DS5.1.1 Groups data to aid analysis and constructs frequency and cumulative frequency tables and graphs (p 116)

Data Analysis and Evaluation

DS5.2.1 Uses the interquartile range and standard deviation to analyse data (p 117)

Data **Stage 2**

DS2.1

Gathers and organises data, displays data using tables and graphs, and interprets the results

Key Ideas

Conduct surveys, classify and organise data using tables

Construct vertical and horizontal column graphs and picture graphs

Interpret data presented in tables, column graphs and picture graphs

Knowledge and Skills

Students learn about

- conducting surveys to collect data
- creating a simple table to organise data

  eg

Red	Blue	Yellow	Green
5	2	7	1

- interpreting information presented in simple tables
- constructing vertical and horizontal column graphs and picture graphs on grid paper using one-to-one correspondence
- marking equal spaces on axes, labelling axes and naming the display
- interpreting information presented in column graphs and picture graphs
- representing the same data in more than one way eg tables, column graphs, picture graphs
- creating a two-way table to organise data

  eg

Drinks	Boys	Girls
Milk	5	6
Water	3	2
Juice	2	1

- interpreting information presented in two-way tables

Working Mathematically

Students learn to

- pose a suitable question to be answered using a survey eg 'What is the most popular playground game among students in our class?' *(Questioning)*
- pose questions that can be answered using the information from a table or graph *(Questioning)*
- create a table to organise collected data, using a computer program eg spreadsheets *(Applying Strategies)*
- use simple graphing software to enter data and create a graph *(Applying Strategies)*
- interpret graphs found on the Internet, in media and in factual texts *(Applying Strategies, Communicating)*
- discuss the advantages and disadvantages of different representations of the same data *(Communicating, Reflecting)*
- compare tables and graphs constructed from the same data to determine which is the most appropriate method of display *(Reasoning)*

Background Information

This topic provides many opportunities for students to collect information about a variety of areas of interest and can be readily linked with other key learning areas such as Human Society and Its Environment (HSIE) and Science.

Data could also be collected from the Internet.

Language

Column graphs consist of vertical columns or horizontal bars. However, the term 'bar graph' is reserved for divided bar graphs and should not be used for a column graph with horizontal bars.

Data	**Stage 3**
DS3.1 Displays and interprets data in graphs with scales of many-to-one correspondence	**Key Ideas** Determine the mean (average) for a small set of data Draw picture, column, line and divided bar graphs using scales of many-to-one correspondence Read and interpret sector (pie) graphs Read and interpret graphs with scales of many-to-one correspondence

Knowledge and Skills

Students learn about

- using the term 'mean' for average
- finding the mean for a small set of data

Picture Graphs and Column Graphs

- determining a suitable scale for data and recording the scale in a key eg ♥= 10 people
- drawing picture or column graphs using a key or scale
- interpreting a given picture or column graph using the key or scale

Line Graphs

- naming and labelling the horizontal and vertical axes
- drawing a line graph to represent any data that demonstrates a continuous change eg hourly temperature
- determining a suitable scale for the data and recording the scale on the vertical axis
- using the scale to determine the placement of each point when drawing a line graph
- interpreting a given line graph using the scales on the axes

Divided Bar Graphs and Sector (Pie) Graphs

- naming a divided bar graph or sector (pie) graph
- naming the category represented by each section
- interpreting divided bar graphs
- interpreting sector (pie) graphs

Working Mathematically

Students learn to

- pose questions that can be answered using the information from a table or graph *(Questioning)*
- collect, represent and evaluate a set of data as part of an investigation, including data collected using the Internet *(Applying Strategies)*
- use a computer database to organise information collected from a survey *(Applying Strategies)*
- use a spreadsheet program to tabulate and graph collected data *(Applying Strategies)*
- determine what type of graph is the best one to display a set of data *(Reflecting)*
- explain information presented in the media that uses the term 'average' eg 'The average temperature for the month of December was 24 degrees.' *(Communicating)*
- discuss and interpret graphs found in the media and in factual texts *(Communicating, Reflecting)*
- identify misleading representations of data in the media *(Reflecting)*
- discuss the advantages and disadvantages of different representations of the same data *(Communicating, Reflecting)*

Background Information

In picture graphs involving numbers that have a large range, one symbol cannot represent one real object.
A key is used for convenience eg ☺ = 10 people.
Line graphs should only be used where meaning can be attached to the points on the line between plotted points.

Sector (pie) graphs and divided bar graphs are used to show how a total is divided into parts.
Column graphs are useful in recording the results obtained from simple probability experiments.
Advantages and disadvantages of different representations of the same data should be explicitly taught.

Data Representation	**Stage 4**
DS4.1 Constructs, reads and interprets graphs, tables, charts and statistical information	**Key Ideas** Draw, read and interpret graphs (line, sector, travel, step, conversion, divided bar, dot plots and stem-and-leaf plots), tables and charts Distinguish between types of variables used in graphs Identify misrepresentation of data in graphs Construct frequency tables Draw frequency histograms and polygons

Knowledge and Skills

Students learn about

- drawing and interpreting graphs of the following types:
  - sector graphs
  - conversion graphs
  - divided bar graphs
  - line graphs
  - step graphs
- choosing appropriate scales on the horizontal and vertical axes when drawing graphs
- drawing and interpreting travel graphs, recognising concepts such as change of speed and change of direction
- using line graphs for continuous data only
- reading and interpreting tables, charts and graphs
- recognising data as quantitative (either discrete or continuous) or categorical
- using a tally to organise data into a frequency distribution table (class intervals to be given for grouped data)
- drawing frequency histograms and polygons
- drawing and using dot plots
- drawing and using stem-and-leaf plots
- using the terms ‘cluster’ and ‘outlier’ when describing data

Working Mathematically

Students learn to

- choose appropriate forms to display data *(Communicating)*
- write a story which matches a given travel graph *(Communicating)*
- read and comprehend a variety of data displays used in the media and in other school subject areas *(Communicating)*
- interpret back-to-back stem-and-leaf plots when comparing data sets *(Communicating)*
- analyse graphical displays to recognise features that may cause a misleading interpretation eg displaced zero, irregular scales *(Communicating, Reasoning)*
- compare the strengths and weaknesses of different forms of data display *(Reasoning, Communicating)*
- interpret data displayed in a spreadsheet *(Communicating)*
- identify when a line graph is appropriate *(Communicating)*
- interpret the findings displayed in a graph eg the graph shows that the heights of all children in the class are between 140 cm and 175 cm and that most are in the group 151–155 cm *(Communicating)*
- generate questions from information displayed in graphs *(Questioning)*

Background Information

The construction of scales on axes can be linked with the drawing of similar figures in Space and Geometry.

It is important that students have the opportunity to gain experience with a wide range of tabulated and graphical data.

Advantages and disadvantages of different representations of the same data should be explicitly taught.

Data may be quantitative (discrete or continuous) or categorical
eg gender (male, female) is categorical
height (measured in cm) is quantitative, continuous
quality (poor, average, good, excellent) is categorical
school population (measured in individuals) is quantitative, discrete.

Language

Students need to be provided with opportunities to discuss what information can be drawn from the data presented. Students need to think about the meaning of the information and to put it into their own words.

Language to be developed would include superlatives, comparatives and other language such as ‘prefer ….over’ etc.

Data Analysis and Evaluation	**Stage 4**
DS4.2 Collects statistical data using either a census or a sample, and analyses data using measures of location and range	**Key Ideas** Use sampling and census Make predictions from samples and diagrams Analyse data using mean, mode, median and range
Knowledge and Skills **Students learn about** • formulating key questions to generate data for a problem of interest • refining key questions after a trial • recognising the differences between a census and a sample • finding measures of location (mean, mode, median) for small sets of data • using a scientific or graphics calculator to determine the mean of a set of scores • using measures of location (mean, mode, median) and the range to analyse data that is displayed in a frequency distribution table, stem-and-leaf plot, or dot plot • collecting data using a random process eg numbers from a page in a phone book, or from a random number function on a calculator • making predictions from a sample that may apply to the whole population • making predictions from a scatter diagram or graph • using spreadsheets to tabulate and graph data • analysing categorical data eg a survey of car colours	**Working Mathematically** **Students learn to** • work in a group to design and conduct an investigation eg - decide on an issue - decide whether to use a census or sample - choose appropriate methods of presenting questions (yes/no, tick a box, a scale of 1 to 5, open-ended, etc) - analyse and present the data - draw conclusions *(Questioning, Reasoning, Applying Strategies, Communicating)* • use spreadsheets, databases, statistics packages, or other technology, to analyse collected data, present graphical displays, and discuss ethical issues that may arise from the data *(Applying Strategies, Communicating, Reflecting)* • detect bias in the selection of a sample *(Applying Strategies)* • consider the size of the sample when making predictions about the population *(Applying Strategies)* • compare two sets of data by finding the mean, mode and/or median, and range of both sets *(Applying Strategies)* • recognise that summary statistics may vary from sample to sample *(Reasoning)* • draw conclusions based on the analysis of data (eg a survey of the school canteen food) using the mean, mode and/or median, and range *(Applying Strategies, Reasoning)* • interpret media reports and advertising that quote various statistics eg media ratings *(Communicating)* • question when it is more appropriate to use the mode or median, rather than the mean, when analysing data *(Questioning)*

Background Information

Many school subjects make use of graphs and data eg in PDHPE students might review published statistics on road accidents, drownings etc.

In Stage 4 Design and Technology, students are required, in relation to marketing, to 'collect information about the needs of consumers in relation to each Design Project'.

The group investigation could relate to aspects of the PDHPE syllabus eg 'appraise the values and attitudes of society in relation to lifestyle and health'.

In Geography, range is used when discussing aspects such as temperature and is given by stating the maximum and minimum values. This is different to the use of 'range' in mathematics where the difference is calculated for the range.

In Geography, use is made of a computer database of local census data. Also, students collect information about global climatic change, greenhouse gas emission, ozone depletion, acid rain, waste management and carbon emissions.

In Science, students carry out investigations to test or research a problem or hypothesis; they collect, record and analyse data and identify trends, patterns and relationships.

Many opportunities occur in this topic to implement aspects of the Key Competencies (see Cross-curriculum Content):
- collecting, analysing and organising information
- communicating ideas and information
- planning and organising activities
- working with others and in teams
- using mathematical ideas and techniques
- solving problems, and
- using technology.

Data Representation and Analysis	**Stage 5.1**
DS5.1.1 Groups data to aid analysis and constructs frequency and cumulative frequency tables and graphs	**Key Ideas** Construct frequency tables for grouped data Find mean and modal class for grouped data Determine cumulative frequency Find median using a cumulative frequency table or polygon
Knowledge and Skills **Students learn about** • constructing a cumulative frequency table for ungrouped data • constructing a cumulative frequency histogram and polygon (ogive) • using a cumulative frequency polygon to find the median • grouping data into class intervals • constructing a frequency table for grouped data • constructing a histogram for grouped data • finding the mean using the class centre • finding the modal class	**Working Mathematically** **Students learn to** • construct frequency tables and graphs from data obtained from different sources (eg the Internet) and discuss ethical issues that may arise from the data *(Applying Strategies, Communicating, Reflecting)* • read and interpret information from a cumulative frequency table or graph *(Communicating)* • compare the effects of different ways of grouping the same data *(Reasoning)* • use spreadsheets, databases, statistics packages, or other technology, to analyse collected data, present graphical displays, and discuss ethical issues that may arise from the data *(Applying Strategies, Communicating, Reflecting)*

Background Information

For grouped data, the mode becomes the 'modal class' and the mean is estimated using the class centre.

Data Analysis and Evaluation	**Stage 5.2**
DS5.2.1 Uses the interquartile range and standard deviation to analyse data	**Key Ideas** Determine the upper and lower quartiles of a set of scores Construct and interpret box-and-whisker plots Find the standard deviation of a set of scores using a calculator Use the terms 'skew' and 'symmetrical' to describe the shape of a distribution
Knowledge and Skills **Students learn about** • determining the upper and lower quartiles for a set of scores • constructing a box-and-whisker plot using the median, the upper and lower quartiles and the extreme values (the 'five-point summary') • finding the standard deviation of a set of scores using a calculator • using the mean and standard deviation to compare two sets of data • comparing the relative merits of measures of spread: - range - interquartile range - standard deviation • using the terms 'skewed' or 'symmetrical' when describing the shape of a distribution	**Working Mathematically** **Students learn to** • compare two or more sets of data using box-and-whisker plots drawn on the same scale *(Applying Strategies)* • compare data with the same mean and different standard deviations (*Applying Strategies*) • compare two sets of data and choose an appropriate way to display these, using back-to-back stem-and-leaf plots, histograms, double column graphs, or box-and-whisker plots *(Communicating, Applying Strategies)* • analyse collected data to identify any obvious errors and justify the inclusion of any scores that differ remarkably from the rest of the data collected *(Applying Strategies, Reasoning)* • use spreadsheets, databases, statistics packages, or other technology, to analyse collected data, present graphical displays, and discuss ethical issues that may arise from the data *(Applying Strategies, Communicating, Reflecting)* • use histograms and stem-and-leaf plots to describe the shape of a distribution (*Communicating)* • recognise when a distribution is symmetrical or skewed, and discuss possible reasons for its shape (*Communicating, Reasoning)*

Background Information

It is intended that students develop a feeling for the concept of standard deviation being a measure of spread of a symmetrical distribution without going into detailed analysis. When using a calculator the σ_n button for standard deviation of a population will suffice.

Use of technology such as computer software and graphics calculators enables 'what if' questions to be asked and explored eg what happens to the standard deviation if a score of zero is added, or if three is added to each score, or if each score is doubled?

Graphics calculators will display box-and-whisker plots for entered data.

No specific analysis of the relative positions of mean, mode and median in skewed distributions is required. Recognition of the general shape and lack of symmetry (only) needs to be considered.

9.5 Measurement

Measurement involves the application of number and geometry understandings and skills when quantifying and solving problems in practical situations. Students need to make reasonable estimates for quantities, be familiar with the most commonly used units for length, area, volume and capacity, and be able to convert between these units. They should develop an idea of the levels of accuracy that are appropriate to particular situations. Competence in applying Pythagoras' theorem to solve practical problems is developed in Stage 4 and applied throughout the topics in Measurement.

While students will develop formulae for the perimeter, area and volume of common shapes, some formulae are not easily derived. In this situation students will need to choose the appropriate formulae and apply them correctly. Application of perimeter, area, surface area and volume skills is extended from simple composite figures in Stage 4 to complex figures by the end of Stage 5.3.

Right-angled triangle trigonometry is introduced in Stage 5.1, with diagrams given, to help students solve practical problems. Emphasis should be on students developing awareness that in right-angled triangles the ratio of sides for a particular angle is constant. Real life applications of trigonometry should be highlighted eg building construction and navigation. Angles of elevation and depression, three-figure bearings and the sixteen compass points (eg NNE) are used in practical problems.

Trigonometry for obtuse angles is introduced in Stage 5.3. Students are not expected to reproduce proofs of the sine, cosine and area rules but these should be done with the students if appropriate.

The Measurement strand for Stages 2 and 3 is organised into five substrands that each focus on a particular attribute:

- Length
- Area
- Volume and Capacity
- Mass
- Time.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Stages 2 and 3 in each substrand. The Stage 4 content is presented in the topics Perimeter and Area, Surface Area and Volume, and Time. The content for Stage 5.1 is represented in the topics Perimeter and Area, and Trigonometry. Stage 5.2 builds on each of these topics as well as Surface Area and Volume. Stage 5.3 builds further on the topics Surface Area and Volume, and Trigonometry.

Summary of Measurement Outcomes for Stages 2 to 5 with page references

Length

MS2.1 Estimates, measures, compares and records lengths, distances and perimeters in metres, centimetres and millimetres (p 120)

MS3.1 Selects and uses the appropriate unit and device to measure lengths, distances and perimeters (p 121)

Area

MS2.2 Estimates, measures, compares and records the areas of surfaces in square centimetres and square metres (p 122)

MS3.2 Selects and uses the appropriate unit to calculate area, including the area of squares, rectangles and triangles (p 123)

Perimeter and Area

MS4.1 Uses formulae and Pythagoras' theorem in calculating perimeter and area of circles and figures composed of rectangles and triangles (p 124)

MS5.1.1 Uses formulae to calculate the area of quadrilaterals and finds areas and perimeters of simple composite figures (p 126)

MS5.2.1 Finds areas and perimeters of composite figures (p 127)

Volume and Capacity

MS2.3 Estimates, measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres (p 128)

MS3.3 Selects and uses the appropriate unit to estimate and measure volume and capacity, including the volume of rectangular prisms (p 130)

Surface Area and Volume

MS4.2 Calculates surface area of rectangular and triangular prisms and volume of right prisms and cylinders (p 131)

MS5.2.2 Applies formulae to find the surface area of right cylinders and volume of right pyramids, cones and spheres and calculates the surface area and volume of composite solids (p 132)

MS5.3.1 Applies formulae to find the surface area of pyramids, right cones and spheres (p 133)

Mass

MS2.4 Estimates, measures, compares and records masses using kilograms and grams (p 134)

MS3.4 Selects and uses the appropriate unit and measuring device to find the mass of objects (p 135)

Time

MS2.5 Reads and records time in one-minute intervals and makes comparisons between time units (p 136)

MS3.5 Uses twenty-four hour time and am and pm notation in real-life situations and constructs timelines (p 137)

MS4.3 Performs calculations of time that involve mixed units (p 138)

Trigonometry

MS5.1.2 Applies trigonometry to solve problems (diagrams given) including those involving angles of elevation and depression (p 139)

MS5.2.3 Applies trigonometry to solve problems including those involving bearings (p 140)

§MS5.3.2 Applies trigonometric relationships, sine rule, cosine rule and area rule in problem solving (p 141)

(§ recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course)

Length	**Stage 2**
MS2.1 Estimates, measures, compares and records lengths, distances and perimeters in metres, centimetres and millimetres	**Key Ideas** Estimate, measure, compare and record lengths and distances using metres, centimetres and/or millimetres Estimate and measure the perimeter of two-dimensional shapes Convert between metres and centimetres, and centimetres and millimetres Record lengths and distances using decimal notation to two places
Knowledge and Skills **Students learn about** • describing one centimetre as one hundredth of a metre • estimating, measuring and comparing lengths or distances using metres and centimetres • recording lengths or distances using metres and centimetres eg 1 m 25 cm • recognising the need for a smaller unit than the centimetre • estimating, measuring and comparing lengths or distances using millimetres • recognising that ten millimetres equal one centimetre and describing one millimetre as one tenth of a centimetre • using the abbreviation for millimetre (mm) • recording lengths or distances using centimetres and millimetres eg 5 cm 3 mm • converting between metres and centimetres, and centimetres and millimetres • recording lengths or distances using decimal notation to two decimal places eg 1.25 m • recognising the features of an object associated with length, that can be measured eg length, breadth, height, perimeter • using the term ‘perimeter’ to describe the total distance around a shape • estimating and measuring the perimeter of two-dimensional shapes • using a tape measure, ruler or trundle wheel to measure lengths or distances	**Working Mathematically** **Students learn to** • describe how a length or distance was measured *(Communicating)* • explain strategies used to estimate lengths or distances eg by referring to a known length *(Communicating, Reflecting)* • select and use an appropriate device to measure lengths or distances *(Applying Strategies)* • question and explain why two students may obtain different measures for the same length, distance or perimeter *(Questioning, Communicating, Reasoning)* • explain the relationship between the size of a unit and the number of units needed eg more centimetres than metres will be needed to measure the same length *(Communicating, Reflecting)*

Background Information

At this Stage, measurement experiences enable students to:
- develop an understanding of the size of the metre, centimetre and millimetre
- estimate and measure using these units, and
- select the appropriate unit and measuring device.

Language

‘Perimeter’ comes from the Greek words that mean to measure around the outside.

Length	**Stage 3**
MS3.1 Selects and uses the appropriate unit and device to measure lengths, distances and perimeters	**Key Ideas** Select and use the appropriate unit and device to measure lengths, distances and perimeters Convert between metres and kilometres; millimetres, centimetres and metres Record lengths and distances using decimal notation to three places Calculate and compare perimeters of squares, rectangles and equilateral and isosceles triangles

Knowledge and Skills

Students learn about

- recognising the need for a unit longer than the metre for measuring distance
- recognising that one thousand metres equal one kilometre and describing one metre as one thousandth of a kilometre
- measuring a kilometre and half-kilometre
- using the abbreviation for kilometre (km)
- converting between metres and kilometres
- measuring and recording lengths or distances using combinations of millimetres, centimetres, metres and kilometres
- converting between millimetres, centimetres and metres to compare lengths or distances
- recording lengths or distances using decimal notation to three decimal places eg 2.753 km
- selecting and using the appropriate unit and device to measure lengths or distances
- interpreting symbols used to record speed in kilometres per hour eg 80 km/h
- finding the perimeter of a large area eg the school grounds
- calculating and comparing perimeters of squares, rectangles and triangles
- finding the relationship between the lengths of the sides and the perimeter for squares, rectangles and equilateral and isosceles triangles

Working Mathematically

Students learn to

- describe how a length or distance was estimated and measured *(Communicating)*
- explain the relationship between the size of a unit and the number of units needed eg more metres than kilometres will be needed to measure the same distance *(Communicating, Reflecting)*
- question and explain why two students may obtain different measures for the same length *(Questioning, Communicating, Reasoning)*
- interpret scales on maps and diagrams to calculate distances *(Applying Strategies, Communicating)*
- solve problems involving different units of length eg Find the total length of three items measuring 5 mm, 20 cm and 1.2 m. *(Applying Strategies)*
- explain that the perimeters of squares, rectangles and triangles can be found by finding the sum of the side lengths *(Communicating, Reasoning)*
- solve simple problems involving speed eg How long would it take to make a journey of 600 km if the average speed for the trip is 75 km/h? *(Applying Strategies)*

Background Information

When the students are able to measure efficiently and effectively using formal units, they should be encouraged to apply their knowledge and skills in a variety of contexts.

Following this they should be encouraged to generalise their method for calculating the perimeter of squares, rectangles and triangles.

Language

'Perimeter' comes from the Greek words that mean to measure around the outside.

Area	**Stage 2**
MS2.2 Estimates, measures, compares and records the areas of surfaces in square centimetres and square metres	**Key Ideas** Recognise the need for square centimetres and square metres to measure area Estimate, measure, compare and record areas in square centimetres and square metres

Knowledge and Skills

Students learn about

- recognising the need for the square centimetre as a formal unit for measuring area
- using a 10 cm × 10 cm tile (or grid) to find areas that are less than, greater than or about the same as 100 square centimetres
- estimating, measuring and comparing areas in square centimetres
- measuring a variety of surfaces using a square centimetre grid overlay
- recording area in square centimetres eg 55 square centimetres
- recognising the need for a unit larger than a square centimetre
- constructing a square metre
- estimating, measuring and comparing areas in square metres
- recording area in square metres eg 5 square metres
- using the abbreviations for square metre (m^2) and square centimetre (cm^2)

Working Mathematically

Students learn to

- question why two students may obtain different measurements for the same area *(Questioning)*
- discuss and compare areas using some mathematical terms *(Communicating)*
- discuss strategies used to estimate area in square centimetres or square metres eg visualising repeated units *(Communicating, Reflecting)*
- apply strategies for measuring the areas of a variety of shapes *(Applying Strategies)*
- use efficient strategies for counting large numbers of square centimetres eg using strips of ten or squares of 100 *(Applying Strategies)*
- explain where square metres are used for measuring in everyday situations eg floor coverings *(Communicating, Reflecting)*
- recognise areas that are 'smaller than', 'about the same as' and 'bigger than' a square metre *(Applying Strategies)*

Background Information

At this Stage, students should appreciate that a formal unit allows for easier and more accurate communication of area measures.

Measurement experiences should enable students to develop an understanding of the size of units, select the appropriate unit, and estimate and measure using the unit.

An important understanding at this Stage is that an area of one square metre need not be a square. It could, for example, be a rectangle, two metres long and half a metre wide.

Language

The abbreviation m^2 is read 'square metre(s)' and not 'metre squared' or 'metre square'.

The abbreviation cm^2 is read 'square centimetre(s)' and not 'centimetre squared' or 'centimetre square'.

Area	**Stage 3**
MS3.2 Selects and uses the appropriate unit to calculate area, including the area of squares, rectangles and triangles	**Key Ideas** Select and use the appropriate unit to calculate area Recognise the need for square kilometres and hectares Develop formulae in words for finding area of squares, rectangles and triangles

Knowledge and Skills	**Working Mathematically**
Students learn about • recognising the need for a unit larger than the square metre • identifying situations where square kilometres are used for measuring area eg a suburb • recognising and explaining the need for a more convenient unit than the square kilometre • measuring an area in hectares eg the local park • using the abbreviations for square kilometre (km^2) and hectare (ha) • recognising that one hectare is equal to 10 000 square metres • selecting the appropriate unit to calculate area • finding the relationship between the length, breadth and area of squares and rectangles • finding the relationship between the base, perpendicular height and area of triangles • reading and interpreting scales on maps and simple scale drawings to calculate an area • finding the surface area of rectangular prisms by using a square centimetre grid overlay or by counting unit squares	**Students learn to** • apply measurement skills to everyday situations eg determining the area of the basketball court *(Applying Strategies)* • use the terms 'length', 'breadth', 'width' and 'depth' appropriately *(Communicating, Reflecting)* • extend mathematical tasks by asking questions eg 'If I change the dimensions of a rectangle but keep the perimeter the same, will the area change?' *(Questioning)* • interpret measurements on simple plans *(Communicating)* • investigate the areas of rectangles that have the same perimeter *(Applying Strategies)* • explain that the area of rectangles can be found by multiplying the length by the breadth *(Communicating, Reasoning)* • explain that the area of squares can be found by squaring the side length *(Communicating, Reasoning)* • equate 1 hectare to the area of a square with side 100 m *(Reflecting)*

Background Information

It is important at this Stage that students establish a real reference for the square kilometre and hectare eg locating a square kilometre or hectare area on a local map.

When the students are able to measure efficiently and effectively using formal units, they should be encouraged to apply their knowledge and skills in a variety of contexts.

Students could be encouraged to find more efficient ways of counting such as finding how many squares in one row and multiplying this by the number of rows.

Students should then begin to generalise their methods to calculate the area of rectangles and triangles. At this Stage, the formulae are described in words and not symbols.

Perimeter and Area	**Stage 4**
MS4.1 Uses formulae and Pythagoras' theorem in calculating perimeter and area of circles and figures composed of rectangles and triangles	**Key Ideas** Describe the limits of accuracy of measuring instruments Develop formulae and use to find the area and perimeter of triangles, rectangles and parallelograms Find the areas of simple composite figures Apply Pythagoras' theorem Investigate and find the area and circumference of circles Convert between metric units of length and area

Knowledge and Skills

Students learn about

Length and Perimeter

- estimating lengths and distances using visualisation strategies
- recognising that all measurements are approximate
- describing the limits of accuracy of measuring instruments ($\pm$ 0.5 unit of measurement)
- interpreting the meaning of the prefixes 'milli', 'centi' and 'kilo'
- converting between metric units of length
- finding the perimeter of simple composite figures

Pythagoras' Theorem

- identifying the hypotenuse as the longest side in any right-angled triangle and also as the side opposite the right angle
- establishing the relationship between the lengths of the sides of a right-angled triangle in practical ways, including the dissection of areas
- using Pythagoras' theorem to find the length of sides in right-angled triangles
- solving problems involving Pythagoras' theorem, giving an exact answer as a surd (eg $\sqrt{5}$) and approximating the answer using an approximation of the square root
- writing answers to a specified or sensible level of accuracy, using the 'approximately equals' sign
- identifying a Pythagorean triad as a set of three numbers such that the sum of the squares of the first two equals the square of the third
- using the converse of Pythagoras' theorem to establish whether a triangle has a right angle

Areas of Squares, Rectangles, Triangles and Parallelograms

- developing and using formulae for the area of a square and rectangle
- developing (by forming a rectangle) and using the formula for the area of a triangle
- finding the areas of simple composite figures that may be dissected into rectangles and triangles

Working Mathematically

Students learn to

- consider the degree of accuracy needed when making measurements in practical situations *(Applying Strategies)*
- choose appropriate units of measurement based on the required degree of accuracy *(Applying Strategies)*
- make reasonable estimates for length and area and check by measuring *(Applying Strategies)*
- select and use appropriate devices to measure lengths and distances *(Applying Strategies)*
- discuss why measurements are never exact *(Communicating, Reasoning)*
- describe the relationship between the sides of a right-angled triangle *(Communicating)*
- use Pythagoras' theorem to solve practical problems involving right-angled triangles *(Applying Strategies)*
- apply Pythagoras' theorem to solve problems involving perimeter and area *(Applying Strategies)*
- identify the perpendicular height of triangles and parallelograms in different orientations *(Communicating)*
- find the dimensions of a square given its perimeter, and of a rectangle given its perimeter and one side length *(Applying Strategies)*
- solve problems relating to perimeter, area and circumference *(Applying Strategies)*
- compare rectangles with the same area and ask questions related to their perimeter such as whether they have the same perimeter *(Questioning, Applying Strategies, Reasoning)*
- compare various shapes with the same perimeter and ask questions related to their area such as whether they have the same area *(Questioning)*
- explain the relationship that multiplying, dividing, squaring and factoring have with the areas of squares and rectangles with integer side lengths *(Reflecting)*
- use mental strategies to estimate the circumference of circles, using an approximate value of π eg 3 *(Applying Strategies)*

Perimeter and Area *(continued)*	**Stage 4**
• developing the formula by practical means for finding the area of a parallelogram eg by forming a rectangle using cutting and folding techniques • converting between metric units of area 1 cm^2 = 100 mm^2 , 1 m^2 = 1 000 000 mm^2 , 1 ha = 10 000 m^2, 1 km^2 = 1 000 000 m^2 = 100 ha ***Circumferences and Areas of Circles*** • demonstrating by practical means that the ratio of the circumference to the diameter of a circle is constant eg by measuring and comparing the diameter and circumference of cylinders • defining the number π as the ratio of the circumference to the diameter of any circle • developing, from the definition of π, formulae to calculate the circumference of circles in terms of the radius r or diameter d $C = \pi d$ or $C = 2\pi r$ • developing by dissection and using the formula to calculate the area of circles $A = \pi r^2$	• find the area and perimeter of quadrants and semi-circles *(Applying Strategies)* • find radii of circles given their circumference or area *(Applying Strategies)* • solve problems involving π , giving an exact answer in terms of π and an approximate answer using $\frac{22}{7}$, 3.14 or a calculator's approximation for π *(Applying Strategies)* • compare the perimeter of a regular hexagon inscribed in a circle with the circle's circumference to demonstrate that $\pi > 3$ *(Reasoning)*

Background Information

This topic links with substitution into formulae in Patterns and Algebra and rounding in Number.

Area and perimeter of quadrants and semicircles is linked with work on fractions.

Graphing of the relationship between a constant perimeter and possible areas of a rectangle is linked with Patterns and Algebra.

Finding the areas of rectangles and squares with integer side lengths is an important link between geometry and multiplication, division, factoring and squares. Factoring a number into the product of two numbers is equivalent to forming a rectangle with these side lengths, and squaring is equivalent to forming a square. Finding perimeters is in turn linked with addition and subtraction.

Students use measurement regularly in Science eg reading thermometers, using measuring cylinders, etc.

Students should develop a sense of the levels of accuracy that are appropriate to a particular situation eg the length of a bridge may be measured in metres to estimate a quantity of paint needed but would need to be measured far more accurately for engineering work.

Area formulae for the triangle and parallelogram need to be developed by practical means and related to the area of a rectangle. The rhombus is treated as a parallelogram and the area found using the formula $A = bh$.

Students should gain an understanding of Pythagoras' theorem, rather than just being able to recite the formula in words. By dissecting and rearranging the squares, they will appreciate that the theorem is a statement of a relationship amongst the areas of squares.

Pythagoras' theorem becomes, in Stage 5, the formula for the circle in the coordinate plane. These links can be developed later in the context of circle geometry and the trigonometry of the general angle.

The number π is known to be irrational (not a fraction) and also transcendental (not the solution of any polynomial equation with integer coefficients). At this Stage, students only need to know that the digits in its decimal expansion do not repeat (all this means is that it is not a fraction), and in fact have no known pattern.

The formula for area of a circle may be established by using one or both of the following dissections:

- cut the circle into a large number of sectors, and arrange them alternately point-up and point-down to form a rectangle with height r and base length πr
- inscribe a number of congruent triangles in a circle, all with vertex at the centre and show that the area of the inscribed polygon is half the length of perimeter times the perpendicular height
- dissect the circle into a large number of concentric rings, cut the circle along a radius, and open it out to form a triangle with height r and base $2\pi r$.

Pythagoras' theorem was probably known many centuries before Pythagoras (c 580-c 500 BC), to at least the Babylonians.

In the 1990s, Wiles finally proved a famous conjecture of Fermat (1601-1665), known as 'Fermat's last theorem', that says that if n is an integer greater than 2, then $a^n + b^n = c^n$ has no integer solution.

The Greek writer, Heron, is best known for his formula for the area of a triangle: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where a, b and c are the lengths of the sides of the triangle and s is half the perimeter of the triangle.

Pi (π) is the Greek letter equivalent to 'p', and is the first letter of the Greek word 'perimetron' meaning perimeter. In 1737, Euler used the symbol for pi for the ratio of the circumference to the diameter of a circle.

One of the three famous problems left unsolved by the ancient Greek mathematicians was the problem of 'squaring the circle' ie using straight edge and compasses to construct a square of area equal to a given circle.

Perimeter and Area	**Stage 5.1**
MS5.1.1 Uses formulae to calculate the area of quadrilaterals and finds areas and perimeters of simple composite figures	**Key Ideas** Develop formulae and use to find the area of rhombuses, trapeziums and kites Find the area and perimeter of simple composite figures consisting of two shapes including quadrants and semicircles

Knowledge and Skills

Students learn about

- developing and using formulae to find the area of quadrilaterals:
  - for a kite or rhombus, $Area = \frac{1}{2}xy$ where x and y are the lengths of the diagonals;
  - for a trapezium, $Area = \frac{1}{2}h(a + b)$ where h is the perpendicular height and a and b the lengths of the parallel sides
- calculating the area of simple composite figures consisting of two shapes including quadrants and semicircles
- calculating the perimeter of simple composite figures consisting of two shapes including quadrants and semicircles

Working Mathematically

Students learn to

- identify the perpendicular height of a trapezium in different orientations *(Communicating)*
- select and use the appropriate formula to calculate the area of a quadrilateral *(Applying Strategies)*
- dissect composite shapes into simpler shapes *(Applying Strategies)*
- solve practical problems involving area of quadrilaterals and simple composite figures *(Applying Strategies)*

Background Information

The formula for finding the area of a rhombus or kite depends upon the fact that the diagonals are perpendicular, and so is linked with the geometry of special quadrilaterals. The formula applies to any quadrilateral in which the diagonals are perpendicular.

The use of formulae links this material with substitution in algebra.

Some practical problems arising from this material are important in consumer arithmetic.

In Geography, students estimate the area of a feature on a map.

Perimeter and Area	**Stage 5.2**
MS5.2.1 Finds areas and perimeters of composite figures	**Key Ideas** Find area and perimeter of more complex composite figures

Knowledge and Skills

Students learn about

- calculating the area and perimeter of sectors
- calculating the perimeter and area of composite figures by dissection into triangles, special quadrilaterals, semicircles and sectors

Working Mathematically

Students learn to

- solve problems involving perimeter and area of composite shapes *(Applying Strategies)*
- calculate the area of an annulus *(Applying Strategies)*
- apply formulae and properties of geometrical shapes to find perimeters and areas eg find the perimeter of a rhombus given the lengths of the diagonals *(Applying Strategies)*
- identify different possible dissections for a given composite figure and select an appropriate dissection to facilitate calculation of the area *(Applying Strategies, Reasoning)*

Background Information

The dissection of composite figures into special quadrilaterals and triangles links with the work on the properties of special shapes in the Space and Geometry strand.

The use of formulae links this material with substitution in algebra.

Some practical problems arising from this material are important in consumer arithmetic.

The area and perimeter of sectors of circles link this material with ratio and fractions.

Volume and Capacity **Stage 2**

MS2.3 - Unit 1 (litres and cubic centimetres)

Estimates, measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres

Key Ideas

Recognise the need for a formal unit to measure volume and capacity

Estimate, measure, compare and record volumes and capacities using litres

Measure the volume of models in cubic centimetres

Knowledge and Skills

Students learn about

- recognising the need for a formal unit to measure volume and capacity
- estimating, measuring and comparing volumes and capacities (to the nearest litre)
- using the abbreviation for litre (L)
- recognising the advantages of using a cube as a unit when packing or stacking
- using the cubic centimetre as a formal unit for measuring volume
- using the abbreviation for cubic centimetre (cm^3)
- constructing three-dimensional objects using cubic centimetre blocks and counting to determine volume
- packing small containers with cubic centimetre blocks and describing packing in terms of layers eg '2 layers of 10 cubic centimetre blocks'

Working Mathematically

Students learn to

- explain the need for a standard unit to measure the volume of liquids and the capacity of containers *(Communicating)*
- estimate the number of cups needed to fill a container with a capacity of one litre *(Applying Strategies)*
- recognise that one litre containers can be a variety of shapes *(Reflecting)*
- relate the litre to familiar everyday containers eg milk cartons *(Reflecting)*
- interpret information about capacity and volume on commercial packaging *(Communicating, Reflecting)*
- estimate the volume of a substance in a partially filled container from the information on the label detailing the contents of the container *(Applying Strategies)*
- distinguish between mass and volume eg 'This stone is heavier than the ball but it takes up less room.' *(Reflecting)*

Background Information

At this Stage, students should appreciate that a formal unit allows for easier and more accurate communication of measures and are introduced to the litre, cubic centimetre and millilitre.

Measurement experiences should enable students to develop an understanding of the size of the unit, estimate and measure using the unit, and select the appropriate unit and measuring device.

Fluids are commonly measured in litres and millilitres. Hence the capacities of containers used to hold fluids are usually measured in litres and millilitres eg a litre of milk will fill a container whose capacity is 1 litre.

The *cubic centimetre* can be introduced and related to the *centimetre* as a unit to measure length and the *square centimetre* as a unit to measure area.

Language

The abbreviation cm^3 is read 'cubic centimetre(s)' and not 'centimetre cubed'.

Volume and Capacity	**Stage 2**
MS2.3 - Unit 2 (millilitres and displacement) Estimates, measures, compares and records volumes and capacities using litres, millilitres and cubic centimetres	**Key Ideas** Estimate, measure, compare and record volumes and capacities using litres and millilitres Convert between litres and millilitres
Knowledge and Skills **Students learn about** • recognising the need for a unit smaller than the litre • estimating, measuring and comparing volumes and capacities using millilitres • making a measuring device calibrated in multiples of 100 millilitres • using a measuring device calibrated in millilitres eg medicine glass, measuring cylinder • using the abbreviation for millilitre (mL) • recognising that 1000 millilitres equal one litre • converting between millilitres and litres eg 1250 mL = 1 litre 250 millilitres • comparing the volumes of two or more objects by marking the change in water level when each is submerged in a container • measuring the overflow in millilitres when different objects are submerged in a container filled to the brim with water	**Working Mathematically** **Students learn to** • explain the need for a standard unit to measure the volume of liquids and the capacity of containers *(Communicating)* • estimate and measure quantities to the nearest 100 mL and/or to the nearest 10 mL *(Applying Strategies)* • interpret information about capacity and volume on commercial packaging *(Communicating, Reflecting)* • estimate the volume of a substance in a partially filled container from the information on the label detailing the contents of the container *(Applying Strategies)* • relate the millilitre to familiar everyday containers and familiar informal units eg 1 teaspoon is approximately 5 mL, 250 mL fruit juice containers *(Reflecting)* • estimate the change in water level expected when an object is submerged *(Applying Strategies)*
Background Information The displacement strategy for finding the volume of an object relies on the fact that an object displaces its own volume when it is totally submerged in a liquid.	The strategy may be applied in two ways - using a partially filled, calibrated, clear container and noting the change in the level of the liquid when the object is submerged, or - submerging an object into a container filled to the brim with liquid and measuring the overflow.
Language The abbreviation cm^3 is read 'cubic centimetre(s)' and not 'centimetres cubed'.	

Volume and Capacity	**Stage 3**
MS3.3 Selects and uses the appropriate unit to estimate and measure volume and capacity, including the volume of rectangular prisms	**Key Ideas** Recognise the need for cubic metres Estimate and measure the volume of rectangular prisms Select the appropriate unit to measure volume and capacity Determine the relationship between cubic centimetres and millilitres Record volume and capacity using decimal notation to three decimal places
Knowledge and Skills **Students learn about** • constructing rectangular prisms using cubic centimetre blocks and counting to determine volume • estimating then measuring the capacity of rectangular containers by packing with cubic centimetre blocks • recognising the need for a unit larger than the cubic centimetre • using the cubic metre as a formal unit for measuring larger volumes • using the abbreviation for cubic metre (m^3) • estimating the size of a cubic metre, half a cubic metre and two cubic metres • selecting the appropriate unit to measure volume and capacity • using repeated addition to find the volume of rectangular prisms • finding the relationship between the length, breadth, height and volume of rectangular prisms • calculating the volume of rectangular prisms • demonstrating that a cube of side 10 cm will displace 1 L of water • demonstrating, by using a medicine cup, that a cube of side 1 cm will displace 1 mL of water • equating 1 cubic centimetre to 1 millilitre and 1000 cubic centimetres to 1 litre • finding the volume of irregular solids in cubic centimetres using a displacement strategy • recording volume and capacity using decimal notation to three decimal places eg 1.275 L	**Working Mathematically** **Students learn to** • explain the advantages of using a cube as a unit to measure volume (*Communicating, Reasoning)* • explain that objects with the same volume may have different shapes *(Communicating, Reflecting)* • construct different rectangular prisms that have the same volume *(Applying Strategies)* • recognise that an object that displaces 300 mL of water has a volume of 300 cubic centimetres *(Reflecting)* • explain why volume is measured in cubic metres in certain situations eg wood bark, concrete *(Communicating, Reasoning)* • estimate the number of cubic metres in a variety of objects such as a cupboard, a car, a bus, the classroom *(Applying Strategies)* • explain that the volume of rectangular prisms can be found by finding the number of cubes in one layer and multiplying by the number of layers *(Applying Strategies, Reflecting)*

Background Information

Volume refers to the space occupied by an object or substance. Capacity refers to the amount a container can hold. Capacity is only used in relation to containers.
It is not necessary to refer to these definitions with students.

When the students are able to measure efficiently and effectively using formal units, they could use centimetre cubes to construct rectangular prisms, counting the number of cubes to determine volume and then begin to generalise their method for calculating the volume.

The *cubic metre* can be introduced and related to the *metre* as a unit to measure length and the *square metre* as a unit to measure area. It is important that students are given opportunities to reflect on their understanding of length and area so they can use this to calculate volume.

Language

The abbreviation m^3 is read 'cubic metre(s)' and not 'metres cubed'.

Surface Area and Volume	**Stage 4**
MS4.2 Calculates surface area of rectangular and triangular prisms and volume of right prisms and cylinders	**Key Ideas** Find the surface area of rectangular and triangular prisms Find the volume of right prisms and cylinders Convert between metric units of volume

Knowledge and Skills

Students learn about

Surface Area of Prisms

- identifying the surface area and edge lengths of rectangular and triangular prisms
- finding the surface area of rectangular and triangular prisms by practical means eg from a net
- calculating the surface area of rectangular and triangular prisms

Volume of Prisms

- converting between units of volume
  1 cm^3 = 1000 mm^3, 1L = 1000 mL = 1000 cm^3,
  1 m^3 = 1000 L = 1 kL
- using the kilolitre as a unit in measuring large volumes
- constructing and drawing various prisms from a given cross-sectional diagram
- identifying and drawing the cross-section of a prism
- developing the formula for volume of prisms by considering the number and volume of layers of identical shape

$$Volume = base\ area \times height$$

- calculating the volume of a prism given its perpendicular height and the area of its cross-section
- calculating the volume of prisms with cross-sections that are rectangular and triangular
- calculating the volume of prisms with cross-sections that are simple composite figures that may be dissected into rectangles and triangles

Volume of Cylinders

- developing and using the formula to find the volume of cylinders (r is the length of the radius of the base and h is the perpendicular height) $V = \pi r^2 h$

Working Mathematically

Students learn to

- solve problems involving surface area of rectangular and triangular prisms *(Applying Strategies)*
- solve problems involving volume and capacity of right prisms and cylinders *(Applying Strategies)*
- recognise, giving examples, that prisms with the same volume may have different surface areas, and prisms with the same surface area may have different volumes *(Reasoning, Applying Strategies)*

Background Information

This outcome is linked with the properties of solids treated in the Space and Geometry strand. It is important that students can visualise rectangular and triangular prisms in different orientations before they find the surface area or volume. They should be able to sketch other views of the object.

The volumes of rectangular prisms and cubes are linked with multiplication, division, factorisation and powers. Factoring a number into the product of three numbers is equivalent to forming a rectangular prism with these side lengths, and to forming a cube if the numbers are all equal. Some students may be interested in knowing what fourth and higher powers, and the product of four or more numbers, correspond to.

When developing the volume formula students require an understanding of the idea of cross-section and can visualise, for example, stacking unit cubes layer by layer into a rectangular prism, or stacking planks into a pile.

The focus here is on right prisms and cylinders, although the formulae for volume also apply to oblique prisms and cylinders provided the perpendicular height is used. Refer to the Background Information in SGS4.1 Properties of Solids (p 148) for definitions of right and oblique prisms and cylinders.

Surface Area and Volume	**Stage 5.2**
MS5.2.2 Applies formulae to find the surface area of right cylinders and volume of right pyramids, cones and spheres and calculates the surface area and volume of composite solids	**Key Ideas** Find surface area of right cylinders and composite solids Find the volume of right pyramids, cones, spheres and composite solids

Knowledge and Skills

Students learn about

Surface Area of Right Cylinders

- developing a formula to find the surface area of right cylinders
  $SA = 2\pi r^2 + 2\pi rh$ for closed cylinders
  where r is the length of the radius and h is the perpendicular height
- finding the surface area of right cylinders
- calculating the surface area of composite solids involving right cylinders and prisms

Volume of Right Pyramids, Cones and Spheres

- using the fact that a pyramid has one-third the volume of a prism with the same base and the same perpendicular height
- using the fact that a cone has one-third the volume of a cylinder with the same base and the same perpendicular height
- using the formula $V = \frac{1}{3}Ah$ to find the volume of pyramids and cones where A is the base area and h is the perpendicular height
- using the formula $V = \frac{4}{3}\pi r^3$ to find the volume of spheres where r is the length of the radius
- finding the dimensions of solids given their volume and/or surface area by substitution into a formula to generate an equation
- finding the volume of prisms whose bases can be dissected into triangles, special quadrilaterals and sectors
- finding the volume of composite solids

Working Mathematically

Students learn to

- solve problems relating to volumes of right pyramids, cones and spheres *(Applying Strategies)*
- dissect composite solids into several simpler solids to find volume *(Applying Strategies)*
- solve practical problems related to volume and capacity eg find the volume of a swimming pool with a given rectangular surface and a given trapezoidal side *(Applying Strategies)*
- solve practical problems related to surface area eg compare the amount of packaging material needed for different shapes *(Applying Strategies)*

Background Information

This work requires a sound understanding of the work in geometry with pyramids, prisms, cones and cylinders.
The use of formulae links this material with substitution into formulae developed in the Patterns and Algebra strand.
Some practical problems arising from this material are important in consumer arithmetic.
The formulae for the volume of solids mentioned here depend only on the perpendicular height and apply equally well to the oblique case. The volume of oblique solids may be included as an extension for some students.
A more systematic development of the volume formulae for spheres, cones and pyramids can be given after integration is developed in Stage 6 (where the factor $\frac{1}{3}$ emerges essentially because the primitive of x^2 is $\frac{1}{3}x^3$).

At this Stage, the relationship could be demonstrated by practical means eg filling a pyramid with sand and pouring into a prism with the same base and perpendicular height and repeating until the prism is filled.
Some students may undertake the following exercise: visualise a cube of side length $2a$ dissected into six congruent pyramids with a common vertex at the centre of the cube, and hence prove that each of these pyramids has volume $\frac{4}{3}a^3$, which is $\frac{1}{3}$ of the enclosing rectangular prism.
The problem of finding the edge length of a cube that has twice the volume of another cube is called ‘the duplication of the cube’, and is one of three famous problems left unsolved by the ancient Greeks. It was proved in the 19th century that this cannot be done with straight edge and compasses, essentially because the cube root of 2 cannot be constructed on the number line.

Surface Area and Volume	**Stage 5.3**
MS5.3.1 Applies formulae to find the surface area of pyramids, right cones and spheres	**Key Ideas** Apply formulae for the surface area of pyramids, right cones and spheres Explore and use similarity relationships for area and volume

Knowledge and Skills

Students learn about

Surface Area of Pyramids, Right Cones and Spheres

- identifying the perpendicular and slant height of pyramids and right cones
- using Pythagoras' theorem to find slant height, base length or perpendicular height of pyramids and right cones
- devising and using methods to calculate the surface area of pyramids
- developing and using the formula to calculate the surface area of cones

  $$\textit{Curved surface area of a cone} = \pi r l$$

  where r is the length of the radius and l is the slant height
- using the formula to calculate the surface area of spheres

  $$\textit{Surface area of a sphere} = 4\pi r^2$$

  where r is the length of the radius
- finding the dimensions of solids given their surface area by substitution into a formula to generate an equation

Similarity, Areas and Volumes

- establishing and applying the fact that in two similar figures with similarity ratio $1: k$
  - matching angles have the same size
  - matching intervals are in the ratio $1: k$
  - matching areas are in the ratio $1: k^2$
  - matching volumes are in the ratio $1: k^3$

Working Mathematically

Students learn to

- apply Pythagoras' theorem to problems involving surface area *(Applying Strategies)*
- solve problems involving the surface area and volume of solids *(Applying Strategies)*
- find surface area of composite solids eg a cylinder with a hemisphere on top *(Applying Strategies)*
- solve problems involving the similarity ratio and areas and volumes *(Applying Strategies)*

Background Information

Pythagoras' theorem is applied here to right-angled triangles in three-dimensional space.

The work here requires a sound knowledge of polyhedra covered in the Space and Geometry strand.

The results concerning ratios of matching areas and volumes in similar three-dimensional figures are linked with work on similar two-dimensional figures in the Space and Geometry strand (see page 156). There is also a link with PDHPE issues such as why babies dehydrate so quickly and why mice eat so much.

The focus in this section is on right cones and right pyramids. Dealing with the oblique version of these objects is difficult and is mentioned only as a possible extension.

The area of the curved surface of a hemisphere is $2\pi r^2$ which is twice the area of its base. This may be a way of making the formula for the surface area of a sphere look reasonable to students. Deriving the relationship between the surface area and the volume of a sphere by dissection into infinitesimal pyramids may be an extension activity for some students. Similarly, some students may investigate as an extension, the surface area of a sphere by projection of infinitesimal squares onto a circumscribed cylinder.

Mass	**Stage 2**
MS2.4 Estimates, measures, compares and records masses using kilograms and grams	**Key Ideas** Recognise the need for a formal unit to measure mass Estimate, measure, compare and record masses using kilograms and grams
Knowledge and Skills **Students learn about** • recognising the need for a formal unit to measure mass • using the kilogram as a unit to measure mass • using hefting to identify objects that are 'more than', 'less than' and 'about the same as' one kilogram • measuring the mass of an object in kilograms using an equal arm balance • estimating and checking the number of similar objects that have a total mass of one kilogram • using the abbreviation for kilogram (kg) • recognising the need for a unit smaller than the kilogram • measuring and comparing the masses of objects in kilograms and grams using a set of scales • using the abbreviation for grams (g) • recognising that 1000 grams equal one kilogram • interpreting commonly used fractions of a kilogram including $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ and relating these to the number of grams	**Working Mathematically** **Students learn to** • recognise that objects with a mass of one kilogram can be a variety of shapes and sizes *(Reflecting)* • interpret statements, and discuss the use of grams and kilograms, on commercial packaging *(Communicating)* • discuss strategies used to estimate mass eg by referring to a known mass *(Communicating)* • question and explain why two students may obtain different measures for the same mass *(Questioning, Communicating, Reasoning)* • solve problems including those involving commonly used fractions of a kilogram *(Applying Strategies)*
Background Information At this Stage, students should appreciate that a formal unit allows for easier and more accurate communication of mass measures and are introduced to the kilogram and gram.	Students should develop an understanding of the size of these units, and estimate and measure using the units.

Mass **Stage 3**

MS3.4

Selects and uses the appropriate unit and measuring device to find the mass of objects

Key Ideas

Recognise the need for tonnes

Convert between kilograms and grams and between kilograms and tonnes

Select and use the appropriate unit and device to measure mass

Record mass using decimal notation to three decimal places

Knowledge and Skills

Students learn about

- choosing appropriate units to measure mass
- recognising the need for a unit larger than the kilogram
- using the tonne to record large masses
  eg sand, soil, vehicles
- using the abbreviation for tonne (t)
- converting between kilograms and grams and between kilograms and tonnes
- selecting and using the appropriate unit and device to measure mass
- recording mass using decimal notation to three decimal places eg 1.325 kg
- relating the mass of one litre of water to one kilogram

Working Mathematically

Students learn to

- solve problems involving different units of mass
  eg find the total mass of three items weighing 50 g, 750 g and 2.5 kg *(Applying Strategies)*
- associate gram measures with familiar objects
  eg a standard egg has a mass of about 60 g *(Communicating)*
- find the approximate mass of a small object by establishing the mass of a number of that object
  eg 'The stated weight of a box of chocolates is 250 g. If there are 20 chocolates in the box, what does each chocolate weigh?' *(Applying Strategies)*

Background Information

Gross mass is the mass of the contents and the container.

Nett mass is the mass of the contents only.

Local industry could provide a source for the study of measurement in tonnes eg weighbridges, cranes and hoists.

Language

'Mass' and' weight' have become interchangeable in everyday usage.

Time	**Stage 2**
MS2.5 Reads and records time in one-minute intervals and makes comparisons between time units	**Key Ideas** Recognise the coordinated movements of the hands on a clock Read and record time using digital and analog notation Convert between units of time Read and interpret simple timetables, timelines and calendars

Knowledge and Skills

Students learn about

- recognising the coordinated movements of the hands on an analog clock, including
  - how many minutes it takes for the minute hand to move from one numeral to the next
  - how many minutes it takes for the minute hand to complete one revolution
  - how many minutes it takes for the hour hand to move from one numeral to the next
  - how many minutes it takes for the minute hand to move from the twelve to any other numeral
  - how many seconds it takes for the second hand to complete one revolution
- associating the numerals 3, 6 and 9 with 15, 30 and 45 minutes and using the terms 'quarter-past' and 'quarter-to'
- identifying which hour has just passed when the hour hand is not pointing to a numeral
- reading analog and digital clocks to the minute eg 7:35 is read as 'seven thirty-five'
- recording digital time using the correct notation eg 9:15
- relating analog notation to digital notation eg ten to nine is the same as 8:50
- converting between units of time

  eg 60 seconds = 1 minute
  60 minutes = 1 hour
  24 hours = 1 day
- reading and interpreting simple timetables, timelines and calendars

Working Mathematically

Students learn to

- recall time facts eg 24 hours in a day *(Communicating, Applying Strategies)*
- discuss time using appropriate language *(Communicating)*
- solve a variety of problems using problem-solving strategies, including:
  - trial and error
  - drawing a diagram
  - working backwards
  - looking for patterns
  - using a table *(Applying Strategies, Communicating)*
- record in words various times as shown on analog and digital clocks *(Communicating)*
- compare and discuss the relationship between time units eg an hour is a longer time than a minute *(Communicating, Reflecting)*

Background Information

Discuss with students the use of informal units of time and their use in other cultures, including the use of Aboriginal time units.

A solar year actually lasts 365 days 5 hours 48 minutes and 45.7 seconds.

Time	**Stage 3**
MS3.5 Uses twenty-four hour time and am and pm notation in real-life situations and constructs timelines	**Key Ideas** Convert between am/pm notation and 24-hour time Compare various time zones in Australia, including during daylight saving Draw and interpret a timeline using a scale Use timetables involving 24-hour time
Knowledge and Skills **Students learn about** • using am and pm notation • telling the time accurately using 24-hour time eg '2330 is the same as 11:30 pm' • converting between 24-hour time and am or pm notation • determining the duration of events using starting and finishing times to calculate elapsed time • using a stopwatch to measure and compare the duration of events • comparing various time zones in Australia, including during daylight saving • reading, interpreting and using timetables from real-life situations, including those involving 24-hour time • determining a suitable scale and drawing a timeline using the scale • interpreting a given timeline using the scale	**Working Mathematically** **Students learn to** • explain where 24-hour time is used eg transport, armed forces, VCRs *(Communicating, Reflecting)* • select the appropriate unit to measure time and order a series of events according to the time taken to complete them *(Applying Strategies)* • determine the local times in various time zones in Australia *(Applying Strategies)* • use bus, train, ferry, and airline timetables, including those accessed on the Internet, to prepare simple travel itineraries *(Applying Strategies)* • use a number of strategies to solve unfamiliar problems, including: - trial and error - drawing a diagram - working backwards - looking for patterns - simplifying the problem - using a table *(Applying Strategies, Communicating)*

Background Information

Australia is divided into three time zones. Time in Queensland, New South Wales, Victoria and Tasmania is Eastern Standard Time (EST), time in South Australia and the Northern Territory is half an hour behind EST, and time in Western Australia is two hours behind EST.

The terms 'am' and 'pm' are used only for the digital form of time recording and not with the 'o'clock' terminology.

The abbreviation *am* stands for the Latin words 'ante meridiem' that means 'before midday'. The abbreviation *pm* stands for 'post meridiem' which means 'after midday'.

Midday and midnight need not be expressed in am or pm form. '12 noon', or '12 midday' and '12 midnight' should be used, even though 12:00 pm and 12:00 am are sometimes seen.

It is important to note that there are many different ways of recording dates, including abbreviated forms. Different notations for dates are used in different countries eg 8th December 2002 is recorded as 8.12.02 in Australia but as 12.8.02 in America.

Time	**Stage 4**
MS4.3 Performs calculations of time that involve mixed units	**Key Ideas** Perform operations involving time units Use international time zones to compare times Interpret a variety of tables and charts related to time
Knowledge and Skills **Students learn about** • adding and subtracting time mentally using bridging strategies eg from 2:45 to 3:00 is 15 minutes and from 3:00 to 5:00 is 2 hours, so the time from 2:45 until 5:00 is 15 minutes + 2 hours = 2 hours 15 minutes • adding and subtracting time with a calculator using the 'degrees, minutes, seconds' button • rounding calculator answers to the nearest minute or hour • interpreting calculator displays for time calculations eg 2.25 on a calculator display for time means $2\frac{1}{4}$ hours • comparing times and calculating time differences between major cities of the world eg 'Given that London is 10 hours behind Sydney, what time is it in London when it is 6:00 pm in Sydney?' • interpreting and using tables relating to time eg tide charts, sunrise/sunset tables, bus, train and airline timetables, standard time zones	**Working Mathematically** **Students learn to** • plan the most efficient journey to a given destination involving a number of connections and modes of transport *(Applying Strategies)* • ask questions about international time relating to everyday life eg whether a particular soccer game can be watched live on television during normal waking hours *(Questioning)* • solve problems involving calculations with mixed time units eg 'How old is a person today if he/she was born on 30/6/1989?' *(Applying Strategies)*

Background Information

Time has links with work on rates involving time eg speed.

The calculation of time can be done on a scientific calculator and links with fractions and decimals.

This topic could be linked to the timing of track and swimming events in the PDHPE syllabus.

The Babylonians thought that the Earth took 360 days to travel around the Sun (last centuries BC). This is why there are 360º in one revolution and hence 90º in one right angle. There are 60 minutes (60') in one hour and 60 minutes in one degree. The word 'minute' (meaning 'small') and minute (time measure), although pronounced differently, are really the same word. A minute (time) is a minute (small) part of one hour. A minute (angle) is a minute (small) part of a right angle.

Trigonometry	**Stage 5.1**
MS5.1.2 Applies trigonometry to solve problems (diagrams given) including those involving angles of elevation and depression	**Key Ideas** Use trigonometry to find sides and angles in right-angled triangles Solve problems involving angles of elevation and angles of depression from diagrams
Knowledge and Skills **Students learn about** ***Trigonometric Ratios of Acute Angles*** • identifying the hypotenuse, adjacent and opposite sides with respect to a given angle in a right-angled triangle in any orientation • labelling the side lengths of a right-angled triangle in relation to a given angle eg the side c is opposite angle C • recognising that the ratio of matching sides in similar right-angled triangles is constant for equal angles • defining the sine, cosine and tangent ratios for angles in right-angled triangles • using trigonometric notation eg sin A • using a calculator to find approximations of the trigonometric ratios of a given angle measured in degrees • using a calculator to find an angle correct to the nearest degree, given one of the trigonometric ratios of the angle ***Trigonometry of Right-Angled Triangles*** • selecting and using appropriate trigonometric ratios in right-angled triangles to find unknown sides, including the hypotenuse • selecting and using appropriate trigonometric ratios in right-angled triangles to find unknown angles correct to the nearest degree • identifying angles of elevation and depression • solving problems involving angles of elevation and depression when given a diagram	**Working Mathematically** **Students learn to** • label sides of right-angled triangles in different orientations in relation to a given angle *(Applying Strategies, Communicating)* • explain why the ratio of matching sides in similar right-angle triangles is constant for equal angles *(Communicating, Reasoning)* • solve problems in practical situations involving right-angled triangles eg finding the pitch of a roof *(Applying Strategies)* • interpret diagrams in questions involving angles of elevation and depression *(Communicating)* • relate the tangent ratio to gradient of a line *(Reflecting)*

Background Information

The definitions of the trigonometric ratios rely on the angle test for similarity, and trigonometry is, in effect, automated calculations with similarity ratios. The topic is thus strongly linked with ratio and with scale drawing.

The fact that the other angles and sides of a right-angled triangle are completely determined by giving two other measurements is justified by the four standard congruence tests.

Trigonometry has practical and analytical applications in surveying, navigation, meteorology, architecture, engineering and electronics.

Trigonometry is introduced through similar triangles with students calculating the ratio of two sides and realising that this remains constant for a given angle.

It is important to emphasise real-life applications of trigonometry eg building construction and surveying.

Emphasis should be placed on correct pronunciation of sin as ‘sine’.

The origin of the word ‘cosine’ is from ‘<u>co</u>mplements <u>sine</u>’, so that cos 40° = sin 50°.

Language

The word trigonometry is derived from two Greek words meaning ‘triangle’ and ‘measurement’.

Trigonometry	**Stage 5.2**
MS5.2.3 Applies trigonometry to solve problems including those involving bearings	**Key Ideas** Solve further trigonometry problems including those involving three-figure bearings
Knowledge and Skills **Students learn about** ***Further Trigonometric Ratios of Acute Angles*** • using a calculator to find trigonometric ratios of a given approximation for angles measured in degrees and minutes • using a calculator to find an approximation for an angle in degrees and minutes, given the trigonometric ratio of the angle ***Further Trigonometry of Right-Angled Triangles*** • finding unknown sides in right-angled triangles where the given angle is measured in degrees and minutes • using trigonometric ratios to find unknown angles in degrees and minutes in right-angled triangles • using three-figure bearings (eg 035º, 225º) and compass bearings eg SSW • drawing diagrams and using them to solve word problems which involve bearings or angles of elevation and depression	**Working Mathematically** **Students learn to** • solve simple problems involving three-figure bearings *(Applying Strategies, Communicating)* • recognise directions given as SSW, NE etc *(Communicating)* • solve practical problems involving angles of elevation and depression *(Applying Strategies)* • check the reasonableness of answers to trigonometry problems *(Reasoning)* • interpret directions given as bearings *(Communicating)* • find the angle between a line with a positive gradient and the x -axis in the coordinate plane by using a right-angled triangle formed by the rise and the run, from the point where the line cuts the x -axis to another point on the line *(Reasoning, Reflecting)*

Background Information

The tangent ratio can be interpreted as the gradient of a line in the coordinate plane.

Students studying circle geometry in the Space and Geometry strand will be able to apply their trigonometry to many problems, making use of the right-angles between a chord and a radius bisecting it, between a tangent and a radius at the point of contact, and in a semicircle.

The work with bearings links to orienteering in PDHPE and map work in Stage 5 Geography.

Students could have practical experience in using clinometers for finding angles of elevation and depression and in using magnetic compasses for bearings.

Students may need encouragement to set out their solutions carefully and to use the correct mathematical language and suitable diagrams.

Students need to recognise the 16 points of a mariner's compass (eg SSW) for comprehension of compass bearings in everyday life eg weather reports.

When setting out their solutions related to finding unknown lengths and angles, students should be advised to give a simplified exact answer eg 25 sin 42º metres or $\sin A = \frac{4}{7}$, then give an approximation correct to a specified or sensible level of accuracy.

Language

The language of bearings needs to be taught explicitly eg the meaning of the word 'of' can be different depending on the question.

§ Trigonometry	Stage 5.3
§ MS5.3.2 Applies trigonometric relationships, sine rule, cosine rule and area rule in problem solving	**Key Ideas** Determine the exact trigonometric ratios for 30°, 45°, 60° Apply relationships in trigonometry for complementary angles and tan in terms of sin and cos Determine trigonometric ratios for obtuse angles Sketch sine and cosine curves Explore trigonometry with non-right-angled triangles: sine rule, cosine rule and area rule Solve problems involving more than one triangle using trigonometry

Knowledge and Skills

Students learn about

Further Trigonometry with Right-Angled Triangles

- proving and using the relationship between the sine and cosine ratios of complementary angles in right-angled triangles

$$\cos A = \sin(90^\circ - A)$$
$$\sin A = \cos(90^\circ - A)$$

- proving that the tangent ratio can be expressed as a ratio of the sine and cosine ratios

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

- determining and using exact sine, cosine and tangent ratios for angles of 30°, 45°, and 60°

The Trigonometric Ratios of Obtuse Angles

- establishing and using the following relationships for obtuse angles, where $0^\circ \le A \le 90^\circ$:

$$\sin(180^\circ - A) = \sin A$$
$$\cos(180^\circ - A) = -\cos A$$
$$\tan(180^\circ - A) = -\tan A$$

- drawing the sine and cosine curves for at least $0^\circ \le A \le 180^\circ$
- finding the possible acute and/or obtuse angles, given a trigonometric ratio

The Sine and Cosine Rules and the Area Rule

- proving the sine rule: In a given triangle ABC, the ratio of a side to the sine of the opposite angle is a constant.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- using the sine rule to find unknown sides and angles of a triangle, including in problems in which there are two possible solutions for an angle
- proving the cosine rule: In a given triangle ABC

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Working Mathematically

Students learn to

- solve problems using exact trigonometric ratios for 30º, 45º and 60º *(Applying Strategies)*
- solve problems, including practical problems, involving the sine and cosine rules and the area rule eg problems related to surveying or orienteering *(Applying Strategies)*
- use appropriate trigonometric ratios and formulae to solve two-dimensional trigonometric problems that require the use of more than one triangle, where the diagram is provided, and where a verbal description is given *(Applying Strategies)*
- recognise that if given two sides and an angle (not included) then two triangles may result, leading to two solutions when the sine rule is applied *(Reasoning, Reflecting, Applying Strategies)*
- explain what happens if the sine, cosine and area rules are applied in right-angled triangles *(Reasoning)*
- ask questions about how trigonometric ratios change as the angle increases from 0° to 180° *(Questioning)*
- recognise that if $\sin A \ge 0$ then there are two possible values for A, given $0^\circ \le A \le 180^\circ$ *(Applying Strategies, Reasoning)*
- find the angle of inclination, θ, of a line in the coordinate plane by establishing and using the relationship gradient = $\tan\theta$ *(Reasoning, Reflecting)*

§ Trigonometry *(continued)*	Stage 5.3
• using the cosine rule to find unknown sides and angles of a triangle • proving and using the area rule to find the area of a triangle: In a given triangle ABC $Area = \frac{1}{2}ab\sin C$ • drawing diagrams and using them to solve word problems that involve non-right-angled triangles	

Background Information

The origin of the word 'cosine' is from 'complements sine', so that cos 40° = sin 50º.

The sine and cosine rules and the area rule are closely linked with the standard congruence tests for triangles. These are the most straightforward ways to proceed:

Given an SAS situation, use the cosine rule to find the third side.

Given an SSS situation, use the alternative form of the cosine rule to find an angle.

Given an AAS situation, use the sine rule to find each unknown side.

Given an ambiguous ASS situation (the angle non-included), use the sine rule to find the sine of the unknown angle opposite the known side - there may then be two solutions for this angle. Alternatively, use the cosine rule to form a quadratic equation for the unknown side.

The cosine rule is a generalisation of Pythagoras' theorem. The sine rule is linked to the circumcircle and to circle geometry.

The definitions of the trigonometric functions in terms of a circle provide the link between Cartesian and polar coordinates. Note that the angle concerned is turned anti-clockwise from the positive x -axis (East). This is not the same as the angle used in navigation (clockwise from North).

The formula gradient = tan θ is a formula for gradient in the coordinate plane.

Circle geometry and the trigonometric functions are closely linked. First, Pythagoras' theorem becomes the equation of a circle in the coordinate plane, and such a circle is used to define the trigonometric functions for general angles. Secondly, the sine and cosine rules are closely linked with the circle geometry theorems concerning angles at the centre and circumference and cyclic quadrilaterals. Many formulae relating the sides, diagonals, angles and area of cyclic quadrilaterals are now accessible.

The trigonometric functions here could be redefined for the general angle using a circle in the coordinate plane - this allows the sine and cosine functions to be plotted for a full revolution and beyond so that their wave nature becomes clear. The intention, however, of this section is for students to become confident using the sine and cosine rules and area rule in practical situations. For many students it is therefore more appropriate to justify the extension of the trigonometric functions to obtuse angles only, either by plotting the graphs and continuing them in the obvious way, or by taking the identities for 180º – θ as definitions. Whatever is done, experimentation with the calculator should be used to confirm this extension.

Students are not expected to reproduce proofs of the sine, cosine and area rules.

Students at this level should realise that when the triangle is right-angled, the cosine rule becomes Pythagoras' theorem, the area formula becomes the simple 'half base times perpendicular height' formula, and the sine rule becomes a simple application of the sine function in a right-angled triangle.

Students studying circle geometry will be able to apply their trigonometry to many problems in circles involving non-right-angled triangles, making use of the supplementary angles at opposite vertices of a cyclic quadrilateral, the equal angles in the same segment, and the alternate segment theorem.

The graphs of the trigonometric functions mark the transition from understanding trigonometry as the study of lengths and angles in triangles (as the word trigonometry implies) to the study of waves, as will be developed in the Stage 6 calculus courses. Waves are fundamental to a vast range of physical and practical phenomena, like light waves and all other electromagnetic waves, and to periodic phenomena like daily temperatures and fluctuating sales over the year, and the major importance of trigonometry lies in the study of these waves.

Brahmagupta, an Indian mathematician and astronomer (c. 598–665 AD), showed that the area of a cyclic quadrilateral is $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, where a, b, c and d are the lengths of the sides of the cyclic quadrilateral and s is the semiperimeter $s = \dfrac{a+b+c+d}{2}$.

This is a generalisation of Heron's formula for the area of a triangle mentioned in MS4.1, as can be seen by putting d=0 so that the quadrilateral becomes a triangle.

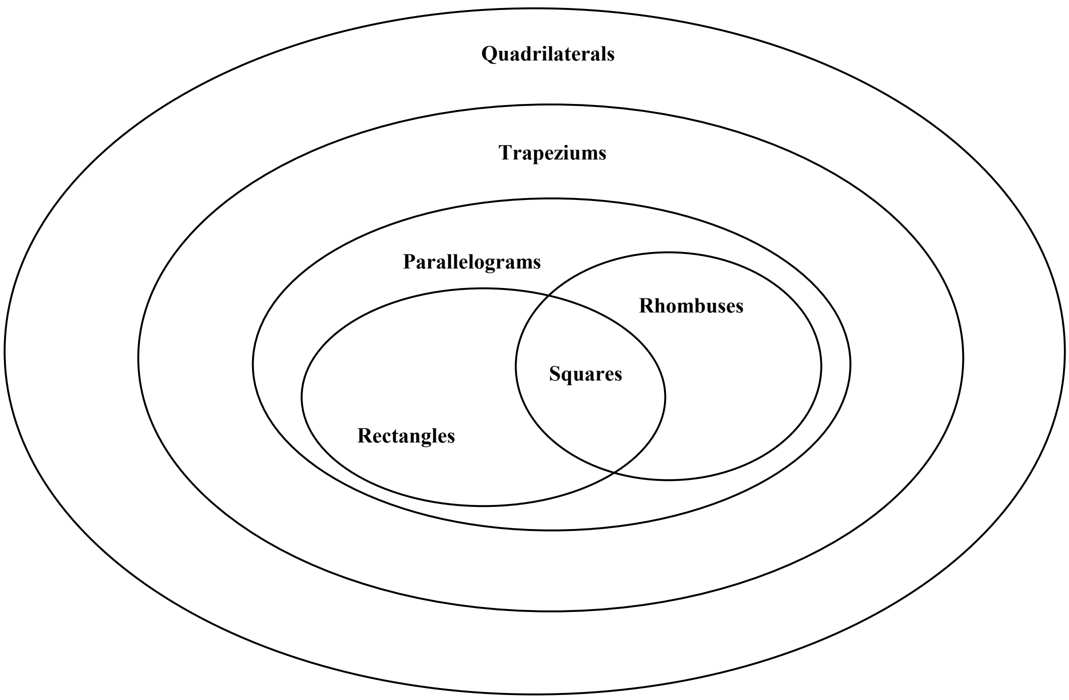
The unit circle is part of the history of trigonometry and explains the derivations of the terms sine, cosine and tangent. A semi-chord in a unit circle subtending at the centre an angle of θ, has length sin θ and is distant cos θ from the centre. The table of values of the sine function was originally called a table of semi-chords. A tangent subtending an angle θ at the centre of a unit circle has length tan θ and hence the name tan for this function.

9.6 Space and Geometry

The Space and Geometry strand enables the investigation of three-dimensional objects and two-dimensional shapes as well as the concepts of position, location and movement. Important and critical skills for students to acquire are those of recognising, visualising and drawing shapes and describing the features and properties of three-dimensional objects and two-dimensional shapes in static and dynamic situations. Features are generally observable whereas properties require mathematical knowledge eg ‘a rectangle has four sides’ is a feature and ‘a rectangle has opposite sides of equal length’ is a property. Manipulation of a variety of real objects and shapes is crucial to the development of appropriate levels of imagery, language and representation.

Geometry uses systematic classification of angles, triangles, regular polygons and polyhedra. The ability to classify is a trait of human cultural development and an important aspect of education. Class inclusivity is a powerful tool in reasoning and determining properties. Justification and reasoning in both an informal and, later, in a formal way are fundamental to geometry in Stages 4 and 5.

When classifying quadrilaterals, students need to begin to develop an understanding of the inclusivity of the classification system. That is, trapeziums are inclusive of the parallelograms, which are inclusive of the rectangles and rhombuses, which are inclusive of the squares. These relationships are presented in the following Venn diagram.



For example, a rectangle is a special type of parallelogram. It is a parallelogram that contains a right angle. A rectangle may also be considered to be a trapezium that has both pairs of opposite sides parallel and equal.

This section presents the outcomes, key ideas, knowledge and skills, and Working Mathematically statements from Stages 2 and 3 in each of the substrands Three-dimensional Space, Two-dimensional Space, and Position. The Stage 4 content is presented in the topics of Properties of Solids, Angles, and Properties of Geometrical Figures. Stage 5.1 does not contain any further Space and Geometry since students need to consolidate the knowledge, skills and understanding developed in Stage 4. The content for Stage 5.2 is presented in the topic Properties of Geometrical Figures and Stage 5.3 is presented in the topic Deductive Geometry, and the optional topic Circle Geometry.

Summary of Space and Geometry Outcomes for Stages 2 to 5 with page references

Three-dimensional Space

SGS2.1 Makes, compares, describes and names three-dimensional objects including pyramids, and represents them in drawings (p 145)

SGS3.1 Identifies three-dimensional objects, including particular prisms and pyramids, on the basis of their properties, and visualises, sketches and constructs them given drawings of different views (p 146)

Properties of Solids

SGS4.1 Describes and sketches three-dimensional solids including polyhedra, and classifies them in terms of their properties (p 147)

Two-dimensional Space

SGS2.2a Manipulates, compares, sketches and names two-dimensional shapes and describes their features (p 149)

SGS2.2b Identifies, compares and describes angles in practical situations (p 150)

SGS3.2a Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties (p 151)

SGS3.2b Measures, constructs and classifies angles (p 152)

Angles

SGS4.2 Identifies and names angles formed by the intersection of straight lines, including those related to transversals on sets of parallel lines, and makes use of the relationships between them (p 153)

Properties of Geometrical Figures

SGS4.3 Classifies, constructs, and determines the properties of triangles and quadrilaterals (p 154)

SGS4.4 Identifies congruent and similar two-dimensional figures stating the relevant conditions (p 156)

SGS5.2.1 Develops and applies results related to the angle sum of interior and exterior angles for any convex polygon (p 157)

SGS5.2.2 Develops and applies results for proving that triangles are congruent or similar (p 158)

§ Deductive Geometry

§SGS5.3.1 Constructs arguments to prove geometrical results (p 159)

§SGS5.3.2 Determines properties of triangles and quadrilaterals using deductive reasoning (p 160)

§SGS5.3.3 Constructs geometrical arguments using similarity tests for triangles (p 162)

 # Circle Geometry

#SGS5.3.4 Applies deductive reasoning to prove circle theorems and to solve problems (p 163)

Position

SGS2.3 Uses simple maps and grids to represent position and follow routes (p 165)

SGS3.3 Uses a variety of mapping skills (p 166)

(# optional topics as further preparation for the Mathematics Extension courses in Stage 6)

(§ recommended topics for students who are following the 5.2 pathway but intend to study the Stage 6 Mathematics course)

Three-dimensional Space **Stage 2**

SGS2.1

Makes, compares, describes and names three-dimensional objects including pyramids, and represents them in drawings

Key Ideas

Name, describe, sort, make and sketch prisms, pyramids, cylinders, cones and spheres

Create nets from everyday packages

Describe cross-sections of three-dimensional objects

Knowledge and Skills

Students learn about

- comparing and describing features of prisms, pyramids, cylinders, cones and spheres
- identifying and naming three-dimensional objects as prisms, pyramids, cylinders, cones and spheres
- recognising similarities and differences between prisms, pyramids, cylinders, cones and spheres
- identifying three-dimensional objects in the environment and from drawings, photographs or descriptions
- making models of prisms, pyramids, cylinders, cones and spheres given a three-dimensional object, picture or photograph to view
- sketching prisms, pyramids, cylinders and cones, attempting to show depth
- creating nets from everyday packages eg a cereal box
- sketching three-dimensional objects from different views including top, front and side views
- making and visualising the resulting cut face (plane section) when a three-dimensional object receives a straight cut
- recognising that prisms have a uniform cross-section when the section is parallel to the base
- recognising that pyramids do not have a uniform cross-section

Working Mathematically

Students learn to

- describe three-dimensional objects using everyday language and mathematical terminology *(Communicating)*
- recognise and describe the use of three-dimensional objects in a variety of contexts eg buildings, packaging *(Reflecting, Communicating)*
- compare features of three-dimensional objects and two-dimensional shapes *(Applying Strategies, Reflecting)*
- compare own drawings of three-dimensional objects with other drawings and photographs of three-dimensional objects *(Reflecting)*
- explore, make and describe the variety of nets that can be used to create particular three-dimensional objects *(Applying Strategies, Reasoning, Communicating)*
- draw three-dimensional objects using a computer drawing package, attempting to show depth *(Applying Strategies)*
- visualise and explain the different two-dimensional shapes resulting when an object is cut in different ways

  eg a cylindrical bread roll could be cut through in two different ways to produce a circle or a rectangle *(Communicating, Reasoning)*

Background Information

The formal names for particular prisms and pyramids are not introduced at this Stage. Prisms and pyramids are to be treated as classes to group all prisms and all pyramids. Names for particular prisms or pyramids are introduced in Stage 3.

Prisms have two bases that are the same shape and size. The bases of a prism may be squares, rectangles, triangles or other polygons. The other faces in the net are rectangular if the faces are perpendicular to the base. The base of a prism is the shape of the uniform cross-section, not necessarily the face on which it is resting.

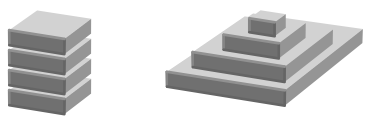
Pyramids differ from prisms in that they have only one base and all the other faces are triangular. The triangular faces meet at a common vertex.

A section is a representation of an object as it would appear if cut by a plane eg if the corner was cut off a cube, the resulting cut face would be a triangle.

An important understanding at this Stage is that the cross-sections parallel to the base of prisms are uniform and the cross-sections parallel to the base of pyramids are not.

Students could explore these ideas by stacking uniform objects to model prisms, and stacking sets of seriated shapes to model pyramids. (Note: such stacks are not strictly pyramids but assist understanding.)

eg



In Geometry a three-dimensional object is called a solid. The three-dimensional object may in fact be hollow but it is still defined as a geometrical solid.

Models at this Stage should include skeletal models.

Three-dimensional Space	**Stage 3**
SGS3.1 Identifies three-dimensional objects, including particular prisms and pyramids, on the basis of their properties, and visualises, sketches and constructs them given drawings of different views	**Key Ideas** Identify three-dimensional objects, including particular prisms and pyramids, on the basis of their properties Construct three-dimensional models given drawings of different views
Knowledge and Skills **Students learn about** • recognising similarities and differences between pyramids or prisms eg between a triangular prism and a hexagonal prism • naming prisms or pyramids according to the shape of their base eg rectangular prism, hexagonal prism • identifying and listing the properties of three-dimensional objects • visualising and sketching three-dimensional objects from different views • constructing three-dimensional models given drawings of different views • visualising and sketching nets for three-dimensional objects • showing simple perspective in drawings by showing depth	**Working Mathematically** **Students learn to** • explain why particular three-dimensional objects are used in the built environment or appear in the natural environment *(Communicating, Reflecting)* • describe to a peer how to construct or draw a three-dimensional object *(Communicating)* • reflect on own drawing of a three-dimensional object and consider whether it can be improved *(Reflecting)* • ask questions about shape properties when identifying them *(Questioning)*

Background Information

At this Stage, the formal names for particular prisms and pyramids (eg rectangular prism, hexagonal pyramid) are introduced while students are engaged in their construction and representation. Only 'family' names were introduced in the previous Stage eg prism.

It is important that geometrical terms are not over-emphasised at the expense of understanding the concepts that the terms represent.

Students at this Stage are continuing to develop their skills of visual imagery, including the ability to

- perceive and hold an appropriate mental image of an object or arrangement, and
- predict the shape of an object that has been moved or altered.

Properties of Solids	**Stage 4**
SGS4.1 Describes and sketches three-dimensional solids including polyhedra, and classifies them in terms of their properties	**Key Ideas** Determine properties of three-dimensional objects Investigate Platonic solids Investigate Euler's relationship for convex polyhedra Make isometric drawings

Knowledge and Skills

Students learn about

- describing solids in terms of their geometric properties
  number of faces
  shape of faces
  number and type of congruent faces
  number of vertices
  number of edges
  convex or non-convex
- identifying any pairs of parallel flat faces of a solid
- determining if two straight edges of a solid are intersecting, parallel or skew
- determining if a solid has a uniform cross-section
- classifying solids on the basis of their properties
  A polyhedron is a solid whose faces are all flat.
  A prism has a uniform polygonal cross-section.
  A cylinder has a uniform circular cross-section.
  A pyramid has a polygonal base and one further vertex (the apex).
  A cone has a circular base and an apex.
  All points on the surface of a sphere are a fixed distance from its centre.
- identifying right prisms and cylinders and oblique prisms and cylinders
- identifying right pyramids and cones and oblique pyramids and cones
- sketching on isometric grid paper shapes built with cubes
- representing three-dimensional objects in two dimensions from different views
- confirming, for various convex polyhedra, Euler's formula
  $$F + V = E + 2$$
  relating the number of faces *(F)*, the number of vertices *(V)* and the number of edges *(E)*
- exploring the history of Platonic solids and how to make them
- making models of polyhedra

Working Mathematically

Students learn to

- interpret and make models from isometric drawings *(Communicating)*
- recognise solids with uniform and non-uniform cross-sections *(Communicating)*
- analyse three-dimensional structures in the environment to explain why they may be particular shapes eg buildings, packaging *(Reasoning)*
- visualise and name a common solid given its net *(Communicating)*
- recognise whether a diagram is a net of a solid *(Communicating)*
- identify parallel, perpendicular and skew lines in the environment *(Communicating, Reflecting)*

Properties of Solids *(continued)* **Stage 4**

Background Information

The volumes, surface areas and edge lengths of solids are a continuing topic of the Measurement strand.

The description above of the cone is not a strict definition unless one adds here, 'and every interval from the apex to a point on the circular edge lies on the curved surface'. For most students it would be inappropriate to raise the issue.

In a right prism, the base and top are perpendicular to the other faces. In a right pyramid or cone, the base has a centre of rotation, and the interval joining that centre to the apex is perpendicular to the base (and thus is its axis of rotation).

Oblique prisms, cylinders, pyramids and cones are those that are not right.

A polyhedron is called regular if its faces are congruent regular polygons and all pairs of adjacent faces make equal angles with each other. There are only five regular polyhedra: the regular tetrahedron, hexahedron (cube), octahedron, dodecahedron and icosahedron. They are also known as the Platonic solids, because Plato used them in his description of the nature of matter. Each can be drawn in a sphere, and a sphere can be drawn inside each.

Polyhedra have three types of boundaries – faces, edges and vertices. Euler's formula gives a relationship amongst the numbers of these boundaries for convex polyhedra. The formula does not always hold if the solid has curved faces or is non-convex.

Students could investigate when and where Plato and Euler lived and their contributions to mathematics.

Students in Years 7–10 Design and Technology may apply the skills developed in this topic, when they 'prepare diagrams, sketches and/or drawings for the making of models or products'.

The Years 7–10 Design and Technology Syllabus, in the section on graphical communication, refers to sketching, drawing with instruments, technical drawing, isometric drawing, orthographic drawing and perspective drawing.

In Science students investigate shapes of crystals. This may involve drawing and building models of crystals.

This topic may be linked to perspective drawing in art work.

Two-dimensional Space **Stage 2**

SGS2.2a

Manipulates, compares, sketches and names two-dimensional shapes and describes their features

Key Ideas

Identify and name pentagons, octagons and parallelograms presented in different orientations

Compare and describe special groups of quadrilaterals

Make tessellating designs by reflecting, translating and rotating

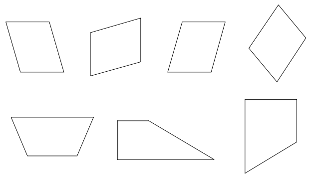
Find all lines of symmetry for a two-dimensional shape

Knowledge and Skills

Students learn about

- manipulating, comparing and describing features of two-dimensional shapes, including pentagons, octagons and parallelograms
- identifying and naming pentagons, octagons, trapeziums and parallelograms presented in different orientations

  eg



- comparing and describing the features of special groups of quadrilaterals
- using measurement to confirm features of two-dimensional shapes eg the opposite sides of a parallelogram are the same length
- grouping two-dimensional shapes using multiple attributes eg those with parallel sides and right angles
- making representations of two-dimensional shapes in different orientations
- constructing two-dimensional shapes from a variety of materials eg plastic, straws and connectors
- comparing the rigidity of two-dimensional frames of three sides with those of four or more sides
- making tessellating designs by reflecting (flipping), translating (sliding) and rotating (turning) a two-dimensional shape
- finding lines of symmetry for a given shape

Working Mathematically

Students learn to

- select a shape from a description of its features *(Applying Strategies, Communicating)*
- describe objects in the environment that can be represented by two-dimensional shapes *(Communicating, Reflecting)*
- explain why a particular two-dimensional shape has a given name eg 'It has four sides, and the opposite sides are parallel.' *(Communicating, Reflecting)*
- recognise that a particular shape can be represented in different sizes and orientations *(Reflecting)*
- use computer drawing tools to create a tessellating design by copying, pasting and rotating regular shapes *(Applying Strategies)*
- describe designs in terms of reflecting, translating and rotating *(Communicating)*
- explain why any line through the centre of a circle is a line of symmetry *(Communicating, Reasoning)*
- determine that a triangle cannot be constructed from three straws if the sum of the lengths of the two shortest straws is less than the longest straw *(Reasoning)*
- explain how four straws of different lengths can produce quadrilaterals of different shapes and also three-dimensional figures *(Communicating, Reasoning)*
- explain why a four-sided frame is not rigid *(Communicating, Reasoning)*

Background Information

It is important for students to experience a variety of shapes in order to develop flexible mental images.
Students need to be able to recognise shapes presented in different orientations. In addition, they should have experiences identifying both regular and irregular shapes. Regular shapes have all sides equal and all angles equal.

When constructing polygons using materials such as straws of different lengths for sides, students should be guided to an understanding that:
- sometimes a triangle cannot be made from 3 straws
- a shape made from three lengths, ie a triangle, is always flat
- a shape made from four or more lengths need not be flat
- a unique triangle is formed if given three lengths
- more than one two-dimensional shape will result if more than three lengths are used.

Language

It is actually the angles that are the focus for the general naming system used for shapes. A polygon (Greek 'many angles') is a closed shape with three or more angles and sides.

Quadrilateral is a term used to describe all four-sided figures.

Two-dimensional Space **Stage 2**

SGS2.2b

Identifies, compares and describes angles in practical situations

Key Ideas

Recognise openings, slopes and turns as angles

Describe angles using everyday language and the term ‘right’

Compare angles using informal means

Knowledge and Skills

Students learn about

- identifying and naming perpendicular lines
- identifying angles with two arms in practical situations eg corners
- identifying the arms and vertex of the angle in an opening, a slope and a turn where one arm is visible
  eg the bottom of a door when it is open is the visible arm and the imaginary line at the base of the doorway is the other arm
- comparing angles using informal means such as an angle tester
- describing angles using everyday language and the term ‘right’ to describe the angle formed when perpendicular lines meet
- drawing angles of various sizes by tracing along the adjacent sides of shapes and describing the angle drawn

Working Mathematically

Students learn to

- identify examples of angles in the environment and as corners of two-dimensional shapes *(Applying Strategies, Reflecting)*
- identify angles in two-dimensional shapes and three-dimensional objects *(Applying Strategies)*
- create simple shapes using computer software involving direction and angles *(Applying Strategies)*
- explain why a given angle is, or is not, a right angle *(Reasoning)*

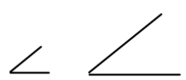
Background Information

At this Stage, students need informal experiences of creating, identifying and describing a range of angles. This will lead to an appreciation of the need for a formal unit to measure angles which is introduced in Stage 3.

The use of informal terms ‘sharp’ and ‘blunt’ to describe acute and obtuse angles respectively are actually counterproductive in identifying the nature of angles as they focus students’ attention to the external points of the angle rather than the amount of turning between the angle arms.

Paper folding is a quick and simple means of generating a wide range of angles for comparison and copying.

A simple angle tester can be created by cutting the radii of two equal circles and sliding the cuts together. Another can be made by joining two narrow straight pieces of card with a split-pin to form the rotatable arms of an angle.



The arms of these angles are different lengths. However, the angles are the same size as the amount of turning between the arms is the same.

Students may mistakenly judge an angle to be greater in size than another on the basis of the length of the arms of the angles in the diagram.

Language

Polygons are named according to the number of angles eg pentagons have five angles, hexagons have six angles, and octagons have eight angles.

Two-dimensional Space	**Stage 3**
SGS3.2a Manipulates, classifies and draws two-dimensional shapes and describes side and angle properties	**Key Ideas** Identify right-angled, isosceles, equilateral and scalene triangles Identify and draw regular and irregular two-dimensional shapes Identify and name parts of a circle Enlarge and reduce shapes, pictures and maps Identify shapes that have rotational symmetry
Knowledge and Skills **Students learn about** • identifying and naming right-angled triangles • manipulating, identifying and naming isosceles, equilateral and scalene triangles • comparing and describing side properties of isosceles, equilateral and scalene triangles • exploring by measurement angle properties of isosceles, equilateral and scalene triangles • exploring by measurement angle properties of squares, rectangles, parallelograms and rhombuses • identifying and drawing regular and irregular two-dimensional shapes from descriptions of their side and angle properties • using templates, rulers, set squares and protractors to draw regular and irregular two-dimensional shapes • identifying and drawing diagonals on two-dimensional shapes • comparing and describing diagonals of different two-dimensional shapes • creating circles by finding points that are equidistant from a fixed point (the centre) • identifying and naming parts of a circle, including the centre, radius, diameter, circumference, sector, semi-circle and quadrant • identifying shapes that have rotational symmetry, determining the order of rotational symmetry • making enlargements and reductions of two-dimensional shapes, pictures and maps • comparing and discussing representations of the same object or scene in different sizes eg student drawings enlarged or reduced on a photocopier	**Working Mathematically** **Students learn to** • select a shape from a description of its features *(Applying Strategies, Communicating)* • describe side and angle properties of two-dimensional shapes *(Communicating)* • construct a shape using computer drawing tools, from a description of its side and angle properties *(Applying Strategies)* • explain classifications of two-dimensional shapes *(Communicating)* • inscribe squares, equilateral triangles, regular hexagons and regular octagons in circles *(Applying Strategies)* • explain the difference between regular and irregular shapes *(Communicating)* • construct designs with rotational symmetry, including using computer drawing tools *(Applying Strategies)* • enlarge or reduce a graphic or photograph using computer software *(Applying Strategies)* • use computer drawing tools to manipulate shapes in order to investigate rotational symmetry *(Applying Strategies)*

Background Information

A shape is said to have rotational symmetry if a tracing of the shape matches it after the tracing is rotated part of a full turn.

Language

Scalene means ‘uneven’ (Greek word ‘skalenos’: uneven): our English word ‘scale’ comes from the same word. Isosceles comes from the two Greek words ‘isos’: equals and ‘skelos’: leg; equilateral comes from the two Latin words ‘aequus’: equal and ‘latus’: side; equiangular comes from ‘aequus’ and another Latin word ‘angulus’: corner.

Two-dimensional Space	**Stage 3**
SGS3.2b Measures, constructs and classifies angles	**Key Ideas** Classify angles as right, acute, obtuse, reflex, straight or a revolution Measure in degrees and construct angles using a protractor
Knowledge and Skills **Students learn about** • identifying the arms and vertex of an angle where both arms are invisible, such as rotations and rebounds • recognising the need for a formal unit for the measurement of angles • using the symbol for degrees (º) • using a protractor to construct an angle of a given size and to measure angles • estimating and measuring angles in degrees • classifying angles as right, acute, obtuse, reflex, straight or a revolution • identifying angle types at intersecting lines	**Working Mathematically** **Students learn to** • describe angles found in their environment *(Communicating, Reflecting)* • compare angles in different two-dimensional shapes *(Applying Strategies)* • explain how an angle was measured *(Communicating)* • rotate a graphic or object through a specified angle about a particular point, including using the rotate function in a computer drawing program *(Applying Strategies)*

Background Information

A circular protractor calibrated from 0° to 360° may be easier for students to use to measure reflex angles than a semicircular protractor calibrated from 0° to 180°.

There are 360° in an angle of complete revolution.

A rebound could be created by rolling a tennis ball towards a wall at an angle and tracing the path with chalk to show the angle.

Angles	**Stage 4**
SGS4.2 Identifies and names angles formed by the intersection of straight lines, including those related to transversals on sets of parallel lines, and makes use of the relationships between them	**Key Ideas** Classify angles and determine angle relationships Construct parallel and perpendicular lines and determine associated angle properties Complete simple numerical exercises based on geometrical properties

Knowledge and Skills

Students learn about

Angles at a Point

- labelling and naming points, lines and intervals using capital letters
- labelling the vertex and arms of an angle with capital letters
- labelling and naming angles using ∠A and ∠XYZ notation
- using the common conventions to indicate right angles and equal angles on diagrams
- identifying and naming adjacent angles (two angles with a common vertex and a common arm), vertically opposite angles, straight angles and angles of complete revolution, embedded in a diagram
- using the words 'complementary' and 'supplementary' for angles adding to 90º and 180º respectively, and the terms 'complement' and 'supplement'
- establishing and using the equality of vertically opposite angles

Angles Associated with Transversals

- identifying and naming a pair of parallel lines and a transversal
- using common symbols for 'is parallel to' (∥) and 'is perpendicular to' (⊥)
- using the common conventions to indicate parallel lines on diagrams
- identifying, naming and measuring the alternate angle pairs, the corresponding angle pairs and the co-interior angle pairs for two lines cut by a transversal
- recognising the equal and supplementary angles formed when a pair of parallel lines are cut by a transversal
- using angle properties to identify parallel lines
- using angle relationships to find unknown angles in diagrams

Working Mathematically

Students learn to

- recognise and explain why adjacent angles adding to 90º form a right angle *(Reasoning)*
- recognise and explain why adjacent angles adding to 180º form a straight angle *(Reasoning)*
- recognise and explain why adjacent angles adding to 360º form a complete revolution *(Reasoning)*
- find the unknown angle in a diagram using angle results, giving reasons *(Applying Strategies, Reasoning)*
- apply angle results to construct a pair of parallel lines using a ruler and a protractor, a ruler and a set square, or a ruler and a pair of compasses *(Applying Strategies)*
- apply angle and parallel line results to determine properties of two-dimensional shapes such as the square, rectangle, parallelogram, rhombus and trapezium *(Applying Strategies, Reasoning, Reflecting)*
- identify parallel and perpendicular lines in the environment *(Reasoning, Reflecting)*
- construct a pair of perpendicular lines using a ruler and a protractor, a ruler and a set square, or a ruler and a pair of compasses *(Applying Strategies)*
- use dynamic geometry software to investigate angle relationships *(Applying Strategies, Reasoning)*

Background Information

At this Stage, students are to be encouraged to give reasons when finding unknown angles. For some students formal setting out could be introduced. For example,

∠ABQ = 70º (corresponding angles, AC ∥ PR)

Eratosthenes' calculation of the circumference of the earth used parallel line results.

Students could explore the results about angles associated with parallel lines cut by a transversal by starting with corresponding angles – move one vertex and all four angles to the other vertex by a translation. The other two results then follow using vertically opposite angles and angles on a straight line. Alternatively, the equality of the alternate angles can be seen by rotation about the midpoint of the transversal.

Properties of Geometrical Figures	**Stage 4**
SGS4.3 Classifies, constructs, and determines the properties of triangles and quadrilaterals	**Key Ideas** Classify, construct and determine properties of triangles and quadrilaterals Complete simple numerical exercises based on geometrical properties

Knowledge and Skills

Students learn about

Notation

- labelling and naming triangles (eg ABC) and quadrilaterals (eg ABCD) in text and on diagrams
- using the common conventions to mark equal intervals on diagrams

Triangles

- recognising and classifying types of triangles on the basis of their properties (acute-angled triangles, right-angled triangles, obtuse-angled triangles, scalene triangles, isosceles triangles and equilateral triangles)
- constructing various types of triangles using geometrical instruments, given different information eg the lengths of all sides, two sides and the included angle, and two angles and one side
- justifying informally by paper folding or cutting, and testing by measuring, that the interior angle sum of a triangle is 180°, and that any exterior angle equals the sum of the two interior opposite angles
- using a parallel line construction, to prove that the interior angle sum of a triangle is 180°
- proving, using a parallel line construction, that any exterior angle of a triangle is equal to the sum of the two interior opposite angles

Quadrilaterals

- distinguishing between convex and non-convex quadrilaterals (the diagonals of a convex quadrilateral lie inside the figure)
- establishing that the angle sum of a quadrilateral is 360°
- constructing various types of quadrilaterals
- investigating the properties of special quadrilaterals (trapeziums, kites, parallelograms, rectangles, squares and rhombuses) by using symmetry, paper folding, measurement and/or applying geometrical reasoning
  Properties to be considered include :
  opposite sides parallel
  opposite sides equal
  adjacent sides perpendicular
  opposite angles equal
  diagonals equal in length
  diagonals bisect each other
  diagonals bisect each other at right angles
  diagonals bisect the angles of the quadrilateral

Working Mathematically

Students learn to

- sketch and label triangles and quadrilaterals from a given verbal description *(Communicating)*
- describe a sketch in sufficient detail for it to be drawn *(Communicating)*
- recognise that a given triangle may belong to more than one class *(Reasoning)*
- recognise that the longest side of a triangle is always opposite the largest angle *(Applying Strategies, Reasoning)*
- recognise and explain why two sides of a triangle must together be longer than the third side *(Applying Strategies, Reasoning)*
- recognise special types of triangles and quadrilaterals embedded in composite figures or drawn in various orientations *(Communicating)*
- determine if particular triangles and quadrilaterals have line and/or rotational symmetry *(Applying Strategies)*
- apply geometrical facts, properties and relationships to solve numerical problems such as finding unknown sides and angles in diagrams *(Applying Strategies)*
- justify their solutions to problems by giving reasons using their own words *(Reasoning)*
- bisect an angle by applying geometrical properties eg constructing a rhombus *(Applying Strategies)*
- bisect an interval by applying geometrical properties eg constructing a rhombus *(Applying Strategies)*
- draw a perpendicular to a line from a point on the line by applying geometrical properties eg constructing an isosceles triangle *(Applying Strategies)*
- draw a perpendicular to a line from a point off the line by applying geometrical properties eg constructing a rhombus *(Applying Strategies)*
- use ruler and compasses to construct angles of 60° and 120° by applying geometrical properties eg constructing an equilateral triangle *(Applying Strategies)*
- explain that a circle consists of all points that are a given distance from the centre and how this relates to the use of a pair of compasses *(Communicating, Reasoning)*
- use dynamic geometry software to investigate the properties of geometrical figures *(Applying Strategies, Reasoning)*

Properties of Geometrical Figures ***(continued)***	**Stage 4**
• investigating the line symmetries and the order of rotational symmetry of the special quadrilaterals • classifying special quadrilaterals on the basis of their properties ***Circles*** • identifying and naming parts of the circle and related lines, including arc, tangent and chord • investigating the line symmetries and the rotational symmetry of circles and of diagrams involving circles, such as a sector and a circle with a chord or tangent	

Background Information

The properties of special quadrilaterals are important in Measurement. For example, the perpendicularity of the diagonals of a rhombus and a kite allow a rectangle of twice the size to be constructed around them, leading to formulae for finding area.

At this Stage, the treatment of triangles and quadrilaterals is still informal, with students consolidating their understandings of different triangles and quadrilaterals and being able to identify them from their properties.

Students who recognise class inclusivity and minimum requirements for definitions may address this Stage 4 outcome concurrently with Stage 5 Space and Geometry outcomes, where properties of triangles and quadrilaterals are deduced from formal definitions.

Students should be encouraged to give reasons orally and in written form for their findings and answers. For some students formal setting out could be introduced.

A range of examples of the various triangles and quadrilaterals should be given, including quadrilaterals containing a reflex angle and figures presented in different orientations.

Mathematical templates and software such as dynamic geometry, and draw and paint packages are additional tools that are useful in drawing and investigating geometrical figures. Computer drawing programs enable students to prepare tessellation designs and to compare these with other designs such as those of Escher.

Language

Scalene means ‘uneven’ (Greek word ‘skalenos’: uneven): our English word ‘scale’ comes from the same word. Isosceles comes from the two Greek words ‘isos’: equals and ‘skelos’: leg; equilateral comes from the two Latin words ‘aequus’: equal and ‘latus’: side; equiangular comes from ‘aequus’ and another Latin word ‘angulus’: corner.

Properties of Geometrical Figures **Stage 4**

SGS4.4	**Key Ideas**
Identifies congruent and similar two-dimensional figures stating the relevant conditions	Identify congruent figures Investigate similar figures and interpret and construct scale drawings

Knowledge and Skills

Students learn about

Congruence

- identifying congruent figures by superimposing them through a combination of rotations, reflections and translations
- matching sides and angles of two congruent polygons
- naming the vertices in matching order when using the symbol $\equiv$ in a congruence statement
- drawing congruent figures using geometrical instruments
- determining the condition for two circles to be congruent (equal radii)

Similarity

- using the term 'similar' for any two figures that have the same shape but not necessarily the same size
- matching the sides and angles of similar figures
- naming the vertices in matching order when using the symbol $|||$ in a similarity statement
- determining that shape, angle size and the ratio of matching sides are preserved in similar figures
- determining the scale factor for a pair of similar polygons
- determining the scale factor for a pair of circles
- calculating dimensions of similar figures using the enlargement or reduction factor
- choosing an appropriate scale in order to enlarge or reduce a diagram
- constructing scale drawings
- drawing similar figures using geometrical instruments

Working Mathematically

Students learn to

- recognise congruent figures in tessellations, art and design work *(Reflecting)*
- interpret and use scales in photographs, plans and drawings found in the media and/or other learning areas *(Applying Strategies, Communicating)*
- enlarge diagrams such as cartoons and pictures *(Applying Strategies)*
- apply similarity to finding lengths in the environment where it is impractical to measure directly eg heights of trees, buildings *(Applying Strategies, Reasoning)*
- apply geometrical facts, properties and relationships to solve problems such as finding unknown sides and angles in diagrams *(Applying Strategies, Reasoning)*
- justify their solutions to problems by giving reasons using their own words *(Reasoning, Communicating)*
- recognise that area, length of matching sides and angle sizes are preserved in congruent figures *(Reasoning)*
- recognise that shape, angle size and the ratio of matching sides are preserved in similar figures *(Reasoning)*
- recognise that similar and congruent figures are used in specific designs, architecture and art work eg works by Escher, Vasarely and Mondrian; or landscaping in European formal gardens *(Reflecting)*
- find examples of similar and congruent figures embedded in designs from many cultures and historical periods *(Reflecting)*
- use dynamic geometry software to investigate the properties of geometrical figures *(Applying Strategies, Reasoning)*

Background Information

Similarity is linked with ratio in the Number strand and with map work in Geography.

Similar and congruent figures are embedded in a variety of designs (eg tapa cloth, Aboriginal designs, Indonesian ikat designs, Islamic designs, designs used in ancient Egypt and Persia, window lattice, woven mats and baskets).

Language

The term 'corresponding' is often used in relation to congruent and similar figures to refer to angles or sides in the same position, but it also has a specific meaning when used to describe a pair of angles in relation to lines cut by a transversal. This syllabus has used 'matching' to describe angles and sides in the same position; however, the use of the word 'corresponding' is not incorrect.

The term 'superimpose' is used to describe the placement of one figure upon another in such a way that the parts of one coincide with the parts of the other.

Properties of Geometrical Figures	**Stage 5.2**
SGS5.2.1 Develops and applies results related to the angle sum of interior and exterior angles for any convex polygon	**Key Ideas** Establish sum of exterior angles result and sum of interior angles result for polygons
Knowledge and Skills **Students learn about** • applying the result for the interior angle sum of a triangle to find, by dissection, the interior angle sum of polygons with 4,5,6,7,8, … sides • defining the exterior angle of a convex polygon • establishing that the sum of the exterior angles of any convex polygon is 360° • applying angle sum results to find unknown angles	**Working Mathematically** **Students learn to** • express in algebraic terms the interior angle sum of a polygon with n sides eg $(n–2) \times 180°$ *(Communicating)* • find the size of the interior and exterior angles of regular polygons with 5,6,7,8, … sides *(Applying Strategies)* • solve problems using angle sum of polygon results *(Applying Strategies)*

Background Information

This topic may be applied when investigating which shapes tessellate.

This work may be extended to interpreting the sum of the exterior angles of a convex polygon as the amount of turning during a circuit of the boundary, and generalising to circles and any closed curve.

Comparing the perimeters of inscribed and circumscribed polygons leads to an approximation for the circumference of a circle. This is the method Archimedes used to develop an approximation for the ratio of the circumference to the diameter, that is, π.

Properties of Geometrical Figures	**Stage 5.2**
SGS5.2.2 Develops and applies results for proving that triangles are congruent or similar	**Key Ideas** Identify similar triangles and describe their properties Apply tests for congruent triangles Use simple deductive reasoning in numerical and non-numerical problems Verify the properties of special quadrilaterals using congruent triangles
Knowledge and Skills **Students learn about** ***Congruent Triangles*** • determining what information is needed to show that two triangles are congruent *If three sides of one triangle are respectively equal to three sides of another triangle, then the two triangles are congruent (SSS).* *If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, then the two triangles are congruent (SAS).* *If two angles and one side of one triangle are respectively equal to two angles and the matching side of another triangle, then the two triangles are congruent (AAS).* *If the hypotenuse and a second side of one right-angled triangle are respectively equal to the hypotenuse and a second side of another right-angled triangle, then the two triangles are congruent (RHS).* • applying the congruency tests to justify that two triangles are congruent • applying congruent triangle results to establish properties of isosceles and equilateral triangles eg *If two sides of a triangle are equal, then the angles opposite the equal sides are equal.* *Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.* *If three sides of a triangle are equal then each interior angle is 60º.* • applying congruent triangle results to establish some of the properties of special quadrilaterals, including diagonal properties eg the diagonals of a parallelogram bisect each other • applying the four triangle congruency tests in numerical exercises to find unknown sides and angles ***Similar Triangles*** • identifying the elements preserved in similar triangles, namely angle size and the ratio of corresponding sides • determining whether triangles are similar • applying the enlargement or reduction factor to find unknown sides in similar triangles • calculating unknown sides in a pair of similar triangles	**Working Mathematically** **Students learn to** • apply the properties of congruent and similar triangles to solve problems, justifying the results *(Applying Strategies, Reasoning)* • apply simple deductive reasoning in solving numerical and non-numerical problems *(Applying Strategies, Reasoning)* • explain why any two equilateral triangles, or any two squares, are similar, and explain when they are congruent *(Communicating, Reasoning)* • investigate whether any two rectangles, or any two isosceles triangles, are similar *(Applying Strategies, Reasoning)* • use dynamic geometry software to investigate the properties of geometrical figures *(Applying Strategies, Reasoning)*
Background Information The definitions of the trigonometric ratios depend upon the similarity of triangles eg any two right-angled triangles in which another angle is 30º must be similar.	 Students are expected to give reasons to justify their results. For some students formal setting out could be introduced. For students proceeding to Stage 5.3 outcomes, this material could be combined with the more formal Euclidean approach in SGS5.3.1 and SGS5.3.2.

§ Deductive Geometry	**Stage 5.3**
§ SGS5.3.1 Constructs arguments to prove geometrical results	**Key Ideas** Construct proofs of geometrical relationships involving congruent or similar triangles
Knowledge and Skills **Students learn about** ***Geometric Reasoning*** • determining minimum conditions to deduce two triangles are congruent • writing formal proofs of congruence of triangles, preserving matching order of vertices • constructing and writing geometrical arguments to prove a general geometrical result, giving reasons at each step of the argument eg prove that the angle in a semicircle is a right angle • proving Pythagoras' theorem and applying it in geometric contexts • applying the converse of Pythagoras' theorem *If the square on one side of a triangle equals the sum of the squares on the other two sides, then the angle between these other two sides is a right angle.* • solving Euclidean geometry problems	**Working Mathematically** **Students learn to** • prove statements about geometrical figures *(Reasoning, Communicating, Applying Strategies)* • solve problems using deductive reasoning *(Reasoning, Applying Strategies)* • make, refine and test conjectures *(Questioning, Communicating, Applying Strategies, Reasoning)* • state possible converses of known results, and examine whether or not they are also true *(Communicating, Applying Strategies, Reasoning)* • use dynamic geometry software to investigate and test conjectures about geometrical figures *(Applying Strategies, Reasoning)*

Background Information

Memorisation of proofs is not intended. Every statement or theorem presented to students to prove could be confirmed first by construction and measurement.

Students may be interested in reading about the history of deductive geometry, including the work of Euclid and Gauss.

Students could investigate who first proved that the angle in a semicircle is a right angle.

It was said that the Academy of Plato (*c* 429–347 BC) had an inscription above its entrance:
ΑΓΕΩΜΕΤΡΗΤΟΣ ΜΗΔΕΙΣ ΕΙΣΙΤΩ literally meaning 'no-one ungeometrical may enter'.

§ Deductive Geometry	**Stage 5.3**
§ SGS5.3.2 Determines properties of triangles and quadrilaterals using deductive reasoning	**Key Ideas** Use deductive geometry to prove properties of special triangles and quadrilaterals
Knowledge and Skills	**Working Mathematically**
Students learn about ***Definitions*** • stating a definition as the minimum amount of information needed to identify a particular figure • proving properties of isosceles and equilateral triangles and special quadrilaterals from the formal definitions of the shapes *<u>Definitions</u>:* *A scalene triangle is a triangle with no two sides equal in length.* *An isosceles triangle is a triangle with two sides equal in length.* *An equilateral triangle is a triangle with all sides equal in length.* *A trapezium is a quadrilateral with at least one pair of opposite sides parallel.* *A parallelogram is a quadrilateral with both pairs of opposite sides parallel.* *A rhombus is a parallelogram with two adjacent sides equal in length.* *A rectangle is a parallelogram with one angle a right angle.* *A square is a rectangle with two adjacent sides equal.* ***Theorems*** • proving and applying theorems and properties related to triangles and quadrilaterals *The sum of the interior angles of a triangle is 180°.* *The exterior angle of a triangle is equal to the sum of the two interior opposite angles.* *If two sides of a triangle are equal, then the angles opposite those sides are equal.* *Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.* *Each angle of an equilateral triangle is equal to 60°.* *The sum of the interior angles of a quadrilateral is 360°.* *The opposite angles of a parallelogram are equal.* *The opposite sides of a parallelogram are equal.* *The diagonals of a parallelogram bisect each other.* *The diagonals of a rhombus bisect each other at right angles.* *The diagonals of a rhombus bisect the vertex angles through which they pass.* *The diagonals of a rectangle are equal.*	**Students learn to** • deduce the size of unknown angles and sides by applying congruency and/or theorems *(Applying Strategies, Reasoning)* • apply theorems and properties of triangles and quadrilaterals to solve problems, justifying solutions using deduction *(Applying Strategies, Reasoning)* • use and interpret formal definitions when presenting a deductive argument *(Applying Strategies, Reasoning, Communicating)* • apply theorems and properties of triangles and quadrilaterals to deduce further geometrical relationships involving triangles and quadrilaterals *(Communicating, Applying Strategies, Reasoning)* • reason that any result proven for a parallelogram would also hold for a rectangle *(Reasoning)* • give reasons why a square is a rhombus, but a rhombus is not necessarily a square *(Reasoning)* • use a diagram or flow chart to show the relationships between different quadrilaterals *(Communicating, Reasoning, Reflecting)* • solve arithmetic and non-arithmetic problems in Euclidean geometry based on known assumptions and proven theorems *(Applying Strategies)* • use dynamic geometry software to investigate and test conjectures about geometrical figures *(Applying Strategies, Reasoning)*

§ Deductive Geometry *(continued)*	**Stage 5.3**
Tests for Quadrilaterals • proving and applying tests for quadrilaterals *If both pairs of opposite angles of a quadrilateral are equal, then it is a parallelogram.* *If both pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram.* *If all sides of a quadrilateral are equal, then it is a rhombus.*	

Background Information

Attention should be given to the logical sequence of theorems and to the types of arguments used. Memorisation of proofs is not intended. Ideally, every theorem presented should be preceded by a straight-edge-and-compasses construction to confirm it, and then proven, in a manner appropriate to the student's work level, by way of an exercise or an investigation.

In Euclidean geometry, congruence is the method by which symmetry arguments are constructed. It is often helpful intuitively to see exactly what transformation, or sequence of transformations, will map one triangle into a congruent triangle. For example, the proof that the opposite sides of a parallelogram are equal involves constructing a diagonal and proving that the resulting triangles are congruent - these two triangles can be transformed into each other by a rotation of 180º about the midpoint of the diagonal.

In the universe of Einstein's general theory of relativity, three-dimensional space is curved, and as a result, the sum of the angles of a physical triangle of cosmological proportions is not 180º. Abstract geometries of this nature were developed by Gauss, Bolyai, Lobachevsky and Riemann and others in the early 19th century, amid suspicions that Euclidean geometry may not be the correct description of physical space.

The Elements of Euclid (c 325-265 BC) gives an account of geometry written almost entirely as a sequence of axioms, definitions, theorems and proofs. Its methods have had an enormous influence on mathematics. Students could read some of Book 1 for a far more systematic account of the geometry of triangles and quadrilaterals.

Dynamic geometry software enables students to explore properties of and relationships between geometrical figures.

§ Deductive Geometry	Stage 5.3
§ SGS5.3.3 Constructs geometrical arguments using similarity tests for triangles	**Key Ideas** Construct geometrical arguments using similarity tests for triangles
Knowledge and Skills **Students learn about** ***Similar Triangles*** • determining what information is needed to establish that two triangles are similar *If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.* *If two sides of one triangle are proportional to two sides of another triangle, and the included angles are equal, then the two triangles are similar.* *If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.* *If the hypotenuse and a second side of a right-angled triangle are proportional to the hypotenuse and a second side of another right-angled triangle, then the two triangles are similar.* • writing formal proofs of similarity of triangles in the standard four- or five-line format, preserving the matching order of vertices, identifying the similarity factor when appropriate, and drawing relevant conclusions from this similarity • proving and applying further theorems using similarity, in particular *The interval joining the midpoints of two sides of a triangle is parallel to the third side and half its length.* *Conversely, the line through the midpoint of a side of a triangle parallel to another side bisects the third side.*	**Working Mathematically** **Students learn to** • prove statements about geometrical figures *(Reasoning, Communicating, Applying Strategies)* • solve problems using deductive reasoning *(Reasoning, Applying Strategies)* • make, refine and test conjectures *(Questioning, Communicating, Applying Strategies, Reasoning)* • state possible converses of known results, and examine whether or not they are also true *(Communicating, Applying Strategies, Reasoning)* • use dynamic geometry software to investigate and test conjectures about geometrical figures *(Applying Strategies, Reasoning)*

Background Information

Work on similar triangles links to Measurement. If the ratio of matching sides is k:1 then the ratio of the areas is k^2 : 1

In the next topic on Circle Geometry, similarity of triangles is used to prove further theorems on intersecting chords, secants and tangents.

 # Circle Geometry **Stage 5.3**

 # SGS5.3.4

Applies deductive reasoning to prove circle theorems and to solve problems

Key Ideas

Deduce chord, angle, tangent and secant properties of circles

Knowledge and Skills

Students learn about

- identifying and naming parts of a circle (centre, radius, diameter, circumference, sector, arc, chord, secant, tangent, segment, semicircle)
- using terminology associated with angles in circles such as subtend, standing on the same arc, angle at the centre, angle at the circumference, angle in a segment
- identifying the arc on which an angle at the centre or circumference stands
- demonstrating that at any point on a circle, there is a unique tangent to the circle, and that this tangent is perpendicular to the radius at the point of contact
- using the above result as an assumption when proving theorems involving tangents
- proving and applying the following theorems:

  Chord Properties

  Chords of equal length in a circle subtend equal angles at the centre and are equidistant from the centre.

  The perpendicular from the centre of a circle to a chord bisects the chord.

  Conversely, the line from the centre of a circle to the midpoint of a chord is perpendicular to the chord.

  The perpendicular bisector of a chord of a circle passes through the centre.

  Given any three non-collinear points, the point of intersection of the perpendicular bisectors of any two sides of the triangle formed by the three points is the centre of the circle through all three points.

  When two circles intersect, the line joining their centres bisects their common chord at right angles.

  Angle properties

  The angle at the centre of a circle is twice the angle at the circumference standing on the same arc.

  The angle in a semicircle is a right angle.

  Angles at the circumference, standing on the same arc, are equal.

  The opposite angles of cyclic quadrilaterals are supplementary.

  An exterior angle at a vertex of a cyclic quadrilateral is equal to the interior opposite angle.

Working Mathematically

Students learn to

- apply circle theorems to prove that the angle in a semicircle is a right angle *(Applying Strategies, Reasoning)*
- apply circle theorems to find unknown angles and sides in diagrams *(Applying Strategies, Reasoning)*
- find the centre of a circle by construction *(Applying Strategies)*
- construct tangents to a circle *(Applying Strategies)*
- use circle and other theorems to prove geometrical results and in problem solving *(Applying Strategies, Reasoning)*

# Circle Geometry *(continued)*	**Stage 5.3**
<u>Tangents and secants</u> *The two tangents drawn to a circle from an external point are equal in length.* *The angle between a tangent and a chord drawn to the point of contact is equal to the angle in the alternate segment.* *When two circles touch, their centres and the point of contact are collinear.* *The products of the intercepts of two intersecting chords of a circle are equal.* *The products of the intercepts of two intersecting secants to a circle from an external point are equal.* *The square of a tangent to a circle from an external point equals the product of the intercepts of any secants from the point.*	

Background Information

As well as solving arithmetic and algebraic problems in circle geometry, students should be able to reason deductively within more theoretical arguments. Diagrams would normally be given to students, with the important information labelled on the diagram to aid reasoning. Students would sometimes need to produce a clear diagram from a set of instructions.

Attention should be given to the logical sequence of theorems and to the types of arguments used. Memorisation of proofs is not intended. Ideally, every theorem presented should be preceded by a straight-edge-and-compasses construction to confirm it, and then proven, in a manner appropriate to the student's work level, by way of an exercise or an investigation.

The tangent-and-radius-theorem is difficult to justify at this Stage, and is probably better taken as an assumption as indicated above.

This topic may be extended to examining the converse of some of the theorems related to cyclic quadrilaterals, leading to a series of conditions for points to be concyclic. However, students may find these results difficult to prove and apply.

The angle in a semicircle theorem is also called Thales' theorem because it was traditionally ascribed to Thales (c 624–548 BC) by the ancient Greeks, who reported that it was the first theorem ever proven in mathematics.

The use of dynamic geometry software enables students to investigate chord, angle, tangent and secant properties.

Position | **Stage 2**

SGS2.3

Uses simple maps and grids to represent position and follow routes

Key Ideas

Use simple maps and grids to represent position and follow routes

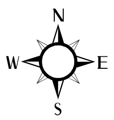
Determine the directions N, S, E and W; NE, NW, SE and SW, given one of the directions

Describe the location of an object on a simple map using coordinates or directions

Knowledge and Skills

Students learn about

- describing the location of an object using more than one descriptor eg 'The book is on the third shelf and second from the left.'
- using a key or legend to locate specific objects
- constructing simple maps and plans eg map of their bedroom
- using given directions to follow a route on a simple map
- drawing and describing a path or route on a simple map or plan
- using coordinates on simple maps to describe position eg 'The lion's cage is at B3.'
- plotting points at given coordinates
- using a compass to find North and hence East, South and West
- using an arrow to represent North on a map
- determining the directions N, S, E and W, given one of the directions
- using N, S, E and W to describe the location of an object on a simple map, given an arrow that represents North eg 'The treasure is east of the cave.'
- using a compass rose to indicate each of the key directions

  eg 

- determining the directions NE, NW, SE and SW, given one of the directions
- using NE, NW, SE and SW to describe the location of an object on a simple map, given a compass rose eg 'The treasure is north-east of the cave.'

Working Mathematically

Students learn to

- use and follow positional and directional language *(Communicating)*
- create simple shapes using computer software involving direction and angles *(Applying Strategies)*
- discuss the use of grids in the environment eg zoo map, map of shopping centre *(Communicating, Reflecting)*
- use computer software involving maps, position and paths *(Applying Strategies)*
- create a simple map or plan using computer paint, draw and graphics tools *(Applying Strategies)*
- use simple coordinates in games, including simulation software *(Applying Strategies)*
- interpret and use simple maps found in factual texts and on the Internet *(Applying Strategies, Communicating)*

Background Information

Grids are used in many contexts to identify position. Students could create their own simple maps and, by drawing a grid over the map, they can then describe locations.

Students need to have experiences identifying North from a compass in their own environment and then determining the other three directions, East, West and South. This could be done in the playground before introducing students to using these directions on maps to describe the positions of various places. The four directions NE, NW, SE and SW could then be introduced to assist with descriptions of places that lie between N, S, E or W.

Position	**Stage 3**
SGS3.3 Uses a variety of mapping skills	**Key Ideas** Interpret scales on maps and plans Make simple calculations using scale
Knowledge and Skills **Students learn about** • finding a place on a map or in a directory, given its coordinates • using a given map to plan or show a route eg route taken to get to the local park • drawing and labelling a grid on a map • recognising that the same location can be represented by maps or plans using different scales • using scale to calculate the distance between two points on a map • locating a place on a map which is a given direction from a town or landmark eg locating a town that is north-east of Broken Hill • drawing maps and plans from an aerial view	**Working Mathematically** **Students learn to** • use coordinates in simulation software and spreadsheets *(Applying Strategies)* • interpret scales on maps and plans *(Applying Strategies, Reflecting)* • give reasons for using a particular scale on a map or plan *(Reasoning)* • use street directories, including those accessed on the Internet, to find the route to a given place *(Applying Strategies)* • describe the direction of one place relative to another eg Perth is west of Sydney *(Applying Strategies, Communicating)*

Background Information

At this Stage, a range of mapping skills could be further developed that include the interpretation of scales and simple calculations to find the actual distance between locations on a map.

This topic links to Human Society and Its Environment (HSIE). These skills could be used to explore the sizes of other countries relative to Australia.

Language

The word ‘scale’ has different meanings in different contexts. Scale could mean the enlargement or reduction factor for a drawing, the scale marked on a measuring device or a fish scale.

10 Life Skills Outcomes and Content

The Board of Studies recognises that a small percentage of students with special education needs may best fulfil the mandatory curriculum requirements for Mathematics Years 7–10 by undertaking Life Skills outcomes and content. (Requirements for access to Life Skills Outcomes and Content are detailed in Section 1.3.)

Life Skills outcomes will be selected on the basis that they meet the particular needs, goals and priorities of each student. Students are not required to complete all outcomes. Outcomes may be demonstrated independently or with support.

In order to provide a relevant and meaningful program of study that reflects the needs, interests and abilities of each student, schools may integrate Mathematics Life Skills outcomes and content across a variety of school and community contexts.

10.1 Outcomes

Objectives	**Outcomes**	
Students will develop:	A student:	
Working Mathematically knowledge, skills and understanding through inquiry, application of problem-solving strategies including the selection and use of appropriate technology, communication, reasoning and reflection.		*Questioning*
	WMLS.1	asks questions about mathematics
		Applying Strategies
	WMLS.2	uses a range of strategies in solving problems
		Communicating
	WMLS.3	responds to and uses mathematical language in everyday situations
		Reasoning
	WMLS.4	checks solutions and reasons to reach conclusions
		Reflecting
	WMLS.5	links their mathematical experiences to everyday life

Students will develop:	A student:	
Number knowledge, skills and understanding in mental and written computation and numerical reasoning.	 NLS.1 NLS.2 NLS.3 NLS.4 NLS.5 NLS.6 NLS.7 NLS.8 NLS.9 NLS.10 NLS.11 NLS.12 NLS.13 NLS.14 NLS.15 NLS.16	*Numeration* recognises language that is descriptive of number counts objects recognises and responds to ordinal terms counts and reads, orders and records numbers recognises fractions in everyday contexts uses fractions in everyday contexts uses decimals in everyday contexts uses percentages in everyday contexts *Operations* uses strategies for addition and subtraction uses strategies for multiplication and division *Money* recognises and matches coins and notes reads and writes amounts of money uses money to purchase goods and services estimates and calculates with money plans personal finances *Chance and Probability* recognises and describes the elements of chance in everyday events
Patterns and Algebra knowledge, skills and understanding in patterning, generalisation and algebraic reasoning.	 PALS.1 PALS.2 PALS.3	*Patterns and Algebra* recognises repeating patterns recognises and continues number patterns calculates missing values by completing simple number sentences
Data knowledge, skills and understanding in collecting, representing, analysing and evaluating information.	 DLS.1 DLS.2	*Data* reads and interprets tables and data displays gathers, organises and displays data

Students will develop:	A student:	
Measurement		
		Time
knowledge, skills and understanding in identifying and quantifying the attributes of shapes and objects and applying measurement strategies.	MLS.1	matches familiar activities with timeframes
	MLS.2	recognises and uses the language of time
	MLS.3	reads and interprets time in a variety of situations
	MLS.4	organises personal time and manages scheduled activities
		Measurement
	MLS.5	responds to the language of measurement in everyday contexts
	MLS.6	uses the language of measurement in everyday contexts
	MLS.7	estimates and measures temperature
	MLS.8	estimates and measures length and distance
	MLS.9	estimates and measures capacity
	MLS.10	estimates and measures mass
	MLS.11	estimates and measures area
Space and Geometry		
		Three-dimensional and two-dimensional space
knowledge, skills and understanding in spatial visualisation and geometric reasoning.	SGLS.1	matches and sorts three-dimensional objects
	SGLS.2	matches and sorts two-dimensional shapes
	SGLS.3	identifies the features of three-dimensional objects and two-dimensional shapes
		Position
	SGLS.4	responds to the language of position
	SGLS.5	uses the language of position in a variety of situations
	SGLS.6	uses simple maps and plans

10.2 Content

The content forms the basis for learning opportunities. Content will be selected on the basis that it meets the needs, goals and priorities for each student. Students are not required to complete all the content to demonstrate achievement of an outcome.

The examples provided are suggestions only.

Number

<table>
<tr><td colspan="2">Numeration</td></tr>
<tr><td colspan="2">NLS.1 A student recognises language that is descriptive of number.</td></tr>
<tr><td>Knowledge and Skills

Students learn about

• recognising language related to number eg none, few, many, more, less</td><td>Working Mathematically

Students learn to

• respond to questions which involve descriptions of number eg ‘Which plate has more cakes?’, ‘Are all the books on the shelf?’, ‘Which box has no pencils?’ (Applying Strategies)
• respond to requests which involve descriptions of number eg ‘Put all the books on the shelf’, ‘Take some paper from my desk’, ‘Place a few chairs near the table’ (Applying Strategies)
• describe and compare groups of objects using language descriptive of number eg ‘There are none left on the shelf’, ‘I have more cards than my brother’ (Applying Strategies, Communicating)</td></tr>
<tr><td colspan="2">NLS.2 A student counts objects.</td></tr>
<tr><td>Knowledge and Skills

Students learn about

• counting objects
• matching groups of objects that have the same number of items
• comparing and ordering groups of objects
• counting objects by twos, fives, tens</td><td>Working Mathematically

Students learn to

• count in meaningful situations eg count out books for a group or class, count uniforms for a sports team (Applying Strategies)
• identify groups that have the same number of items as a given group, more items than a given group or fewer items than a given group (Applying Strategies)
• count objects into equal bundles (Applying Strategies)</td></tr>
</table>

Numeration	
NLS.3 A student recognises and responds to ordinal terms.	
Knowledge and Skills **Students learn about** • recognising ordinal terms eg first, second, third	**Working Mathematically** **Students learn to** • respond to directions involving ordinal terms eg 'Give a ball to the first person in each row', 'Put a book on every second chair' *(Applying Strategies)* • use ordinal terms in meaningful contexts eg 'The youth group meets on the first Monday of each month', 'My birthday is on the fifth of November' *(Communicating)* • ask questions involving directions eg 'Is the ladies toilet on the first or second floor?' 'Is the supermarket the third building on the left?' *(Questioning)*
NLS.4 A student counts and reads, orders and records numbers.	
Knowledge and Skills **Students learn about** • counting and reading, ordering and recording numbers 0–9 • counting and reading, ordering and recording two-digit numbers • recognising and reading numerals in a range of formats • recognising, reading and converting Roman numerals used in everyday contexts • counting and reading, ordering and recording three-digit numbers • counting forwards and backwards from a given number in the range 0–100 • counting by twos, fives, tens and hundreds • recognising odd and even numbers • recognising and reading numbers with more than three digits • reading and recording ordinal terms	**Working Mathematically** **Students learn to** • identify some of the ways numbers are used in our lives eg telephone numbers, bus numbers, PIN *(Reflecting)* • identify and locate numbers in a range of situations (eg seat numbers in a theatre, odd and even house numbers, table of contents in a book) *(Applying Strategies, Reflecting)* • interpret numerical information from text (eg recipes, medication dosages), graphs and tables *(Applying Strategies, Reflecting)* • tell the time on a watch or clock by counting by fives around the clock or watch face *(Applying Strategies, Reflecting)* • ask questions involving counting eg 'How many people are in class today?' *(Questioning)* • write ordinal terms eg 'My birthday is on the 30th of November' *(Communicating)*

Numeration

NLS.5 A student recognises fractions in everyday contexts.

Knowledge and Skills	**Working Mathematically**
Students learn about	**Students learn to**
Half and halves	
• recognising the terms 'half' and 'halves' in everyday situations • sharing an object eg giving half a sandwich	• respond to fraction language in everyday situations eg 'Take half a sandwich' *(Applying Strategies, Reflecting)*
Quarters • recognising the term 'quarter' in everyday situations • sharing an object eg giving a quarter of an apple	• follow an instruction involving fraction language in everyday situations eg 'Give a quarter of the orange to your friend' *(Applying Strategies)* • recognise the use of fractions in everyday contexts, eg half-hour television programs *(Reflecting)*

NLS.6 A student uses fractions in everyday contexts.

Knowledge and Skills	**Working Mathematically**
Students learn about	**Students learn to**
Half and halves • recognising the need for two equal parts when dividing a whole in half • putting two halves together to make a whole eg two half sandwiches is the same as one whole sandwich • halving a group of objects by sharing into two equal piles eg 'If I take half of the six lollies, I will have three lollies' • using fraction notation for a half ($\frac{1}{2}$) • combining a half with whole numbers eg $3\frac{1}{2}$	• allocate portions or divide materials eg cut a cake into equal pieces *(Applying Strategies)* • question if parts of a whole object, or collection of objects, are equal eg 'Has the cake been cut into two equal parts?' *(Questioning)* • identify items that are about a half eg half an apple *(Applying Strategies)* • identify items that are less than a half or more than a half eg 'The glass is more than half full' *(Applying Strategies)*
Quarters • putting four quarters together to make a whole • putting two quarters together to make a half • using fraction notation for quarters eg $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$	• describe situations using the terms 'half' and 'halves' *(Communicating)* • recognise the use of fractions in everyday contexts eg 'Lunch is three-quarters of an hour' *(Communicating, Reflecting)*
Thirds • using the term 'third' in everyday situations • sharing an object by dividing it into three equal parts • using fraction notation for thirds eg $\frac{1}{3}, \frac{2}{3}$	• follow instructions involving the use of 'quarter' and/or 'third' eg 'cut an apple in quarters', 'use $\frac{1}{3}$ of a cup of milk in a recipe' *(Applying Strategies)* • indicate the relative size of a fraction or mixed number eg '$3\frac{1}{2}$ is more than 3 but less than 4' *(Communicating, Reasoning)*

Numeration

NLS.7 A student uses decimals in everyday contexts.

Knowledge and Skills	Working Mathematically
Students learn about	**Students learn to**
• reading numbers as decimals eg 3.5 is read as 'three point five' • adding and subtracting numbers to two decimal places in the context of money eg \$2.25 + \$1.25 • equating fractions to decimals in the context of parts of one dollar eg 1 cent $= \frac{1}{100}$ of \$1 = \$0.01 10 cents $= \frac{10}{100}$ of \$1 = \$0.10 50 cents $= \frac{50}{100}$ of \$1 = \$0.50 • expressing one half ($\frac{1}{2}$) as a decimal (0.5) • interpreting decimal notation for tenths and hundredths eg 0.1 is the same as $\frac{1}{10}$, 0.01 is the same as $\frac{1}{100}$ • recognising commonly used fractions as decimals eg $\frac{1}{2} = \frac{5}{10} = 0.5$ $\frac{1}{4} = \frac{25}{100} = 0.25$ • comparing decimals with the same number of decimal places eg 0.3 is less than 0.5 • rounding decimals in the context of money eg rounding \$4.99 to \$5	• recognise the use of decimals in the timing of races eg timing swimming races to tenths and hundredths of a second *(Reflecting)* • interpret the use of decimals for recording measurements eg 3.5 m means three and a half metres *(Communicating, Applying Strategies)* • recognise that 0.50 is the same as 0.5 *(Reasoning)* • recognise the use of decimals in the community eg advertisements for interest rates *(Reflecting)* • decide which is the best interest rate offered for a loan *(Applying Strategies, Reflecting)* • interpret calculator displays involving decimals *(Applying Strategies)* • explain the result of rounding when purchasing goods where the total number of cents involved cannot be made up using 5 and 10 cent pieces eg \$5.02 becomes \$5.00, \$2.03 becomes \$2.05 *(Communicating, Applying Strategies, Reasoning, Reflecting)*

NLS.8 A student uses percentages in everyday contexts.

Knowledge and Skills	Working Mathematically
Students learn about	**Students learn to**
• reading the symbol % as 'percent' • relating percentages to a fraction eg 25% means 25 out of 100 or $\frac{25}{100}$ • expressing commonly used percentages as a fraction eg $50\% = \frac{50}{100} = \frac{1}{2}$ $25\% = \frac{25}{100} = \frac{1}{4}$ $10\% = \frac{10}{100} = \frac{1}{10}$ • calculating simple percentages of amounts of money eg 50% of \$8 is \$4	• recognise the use of the % symbol across key learning areas and in a variety of contexts eg advertising, discounts *(Reflecting)* • calculate discounts eg 10% off \$50 *(Applying Strategies)* • solve problems involving percentages using a calculator *(Applying Strategies)* • interpret advertising and media reports involving percentages eg 90% success rate for goal kicking, 25% more chocolate *(Communicating, Applying Strategies, Reflecting)*

Operations	
NLS.9 A student uses strategies for addition and subtraction.	
Knowledge and Skills	**Working Mathematically**
Students learn about • combining two or more groups of objects to model addition • separating and taking part of a group of objects away to model subtraction • comparing two groups of objects to determine 'how many more' • creating combinations for numbers to at least 10 eg 'How many more to make ten?' • counting forwards by ones to add and backwards by ones to subtract • using the terms 'add', 'plus', 'equals', 'is equal to', 'take away', 'minus' and 'the difference between' • recognising and using the symbols + , – and = • adding two numbers using concrete materials, mental strategies, written processes and/or calculator strategies • adding more than two numbers using mental, written and/or calculator strategies eg 6 + 2 + 5 • subtracting a number from a given number using concrete materials, mental strategies, written processes and/or calculator strategies	**Students learn to** • estimate the sum of two numbers and check by writing the calculation or using a calculator *(Applying Strategies)* • calculate the change expected when purchasing an item eg change from $1 when purchases total 65 cents *(Applying Strategies, Reflecting)* • solve problems involving the addition and/or the subtraction of sums of money *(Applying Strategies, Reflecting)* • apply their strategies for addition in game situations *(Applying Strategies)*

Operations

NLS.10 A student uses strategies for multiplication and division.

Knowledge and Skills	**Working Mathematically**
Students learn about • modelling multiplication using concrete materials eg three cups each containing four spoons • modelling multiplication using arrays ● ● ● ● 3 rows of 4 equals 12 ● ● ● ● $4 + 4 + 4 = 12$ ● ● ● ● $3 \times 4 = 12$ • multiplying two numbers using concrete materials, mental strategies, written processes and/or calculator strategies • modelling division by sharing concrete materials eg share 12 lollies among 3 students • modelling division using repeated subtraction with concrete materials eg 'How many bags of lollies can I make from a total of 20 lollies if each bag is to contain 5 lollies?' • dividing numbers using concrete materials, mental strategies, written processes and/or calculator strategies	**Students learn to** • determine the total cost of a number of items given the price of one *(Applying Strategies)* • determine quantities needed when preparing a meal for several people using a recipe based on ingredients for one person *(Applying Strategies)* • share a number of items between two or more people using concrete materials *(Applying Strategies)* • calculate the cost of a single item when given the price of a pack containing several items *(Applying Strategies)* • solve problems involving multiplication and/or division eg 'How many cars are needed to transport 12 students if each car can take 3 students together with the driver?' *(Applying Strategies)* • apply multiplication and/or division strategies in situations from other key learning areas eg 'How many laps of a 50 metre pool are needed to swim a total of 200 metres?' *(Applying Strategies, Reflecting)*

Money

NLS.11 A student recognises and matches coins and notes.

Knowledge and Skills	**Working Mathematically**
Students learn about • recognising a range of coins and notes • matching and sorting coins and notes into groups on the basis of face value • ordering coins and notes on the basis of face value • recognising that coins and notes have different values	**Students learn to** • indicate the appropriate coin to purchase a specific item in the school canteen eg 50 cent coin, $2 coin *(Applying Strategies)* • indicate the most appropriate note to purchase an item in a shop eg five dollar note, twenty dollar note *(Applying Strategies)*

Money	
NLS.12 A student reads and writes amounts of money.	
Knowledge and Skills **Students learn about** • recognising the cost of goods or services eg read price on tags attached to clothing, identify the cost of items in a supermarket as indicated on the shelf, read a notice board at a theatre to determine the price of entry • writing amounts in cents • writing amounts in dollars • writing amounts of money using decimal notation • writing amounts of money in words	**Working Mathematically** **Students learn to** • identify the cost of items up to $10 in value by locating prices eg a drink at the school canteen is $1.00, a magazine at the supermarket is $3.75 *(Communicating, Applying Strategies, Reflecting)* • identify the cost of items up to $100 in value by locating prices eg a meal at a restaurant is $22, a jacket is $80, a pair of sunglasses is $99.95 *(Communicating, Applying Strategies, Reflecting)* • write amounts of money involving cents, dollars, and combinations of dollars and cents eg 25c, $5, $4.75, $89.95 *(Applying Strategies)* • complete a cheque using words and decimal notation *(Applying Strategies, Reflecting, Communicating)*
NLS.13 A student uses money to purchase goods and services.	
Knowledge and Skills **Students learn about** • recognising that money has value • recognising that money is a medium for obtaining goods and services • recognising the hierarchy of value attached to goods and services • counting coins of the same denomination • counting coins of different denomination • counting notes of the same denomination • counting notes of different denomination • matching a range of coins to demonstrate equivalence of value eg 2×20 cent coins and 1×10 cent coin is equivalent to a 50 cent coin, 6×5 cent coins is equivalent to 3×10 cent coins • matching a range of notes to demonstrate equivalence of value eg $2 \times \$5$ notes and $1 \times \$10$ note is equivalent to a $20 note	**Working Mathematically** **Students learn to** • use coins or notes to pay for purchases eg to buy lunch in the school canteen, to buy a ticket at the cinema, to pay for items at a supermarket *(Applying Strategies)* • use coins or notes to pay for services eg pay for dry cleaning, pay a taxi fare, pay for a hair cut *(Applying Strategies)* • tender an amount of money using a combination of coins and notes eg purchase items to the value of $5 by tendering $2 \times \$2$ coins and $1 \times \$1$ coin, purchase items to the value of $20 by tendering $1 \times \$10$ note, $1 \times \$5$ note and $5 \times \$1$ coins. *(Applying Strategies, Reasoning)* • determine if they have enough money to pay for a particular item or service *(Applying Strategies, Reasoning)* • use the language of money in a range of contexts *(Communicating)* • check the details of purchases on receipts or dockets *(Reflecting, Applying Strategies)*

<table>
<tr><td colspan="2">Money</td></tr>
<tr><td colspan="2">NLS.14 A student estimates and calculates with money.</td></tr>
<tr><td>Knowledge and Skills

Students learn about

• estimating amounts of money to tender for goods or services
• calculating amounts of money to tender for goods or services
• estimating the amount of change due in relation to a transaction for goods or services
• calculating the amount of change due in relation to a transaction for goods or services
• calculating the amount of time it will take to save for items at a specific rate per week or month</td><td>Working Mathematically

Students learn to

• estimate the cost of a range of items and select the appropriate coin or note to pay for the items eg selects a $2 coin to pay for sweets at a milk bar, selects a $20 note to pay for a T-shirt at a discount store, estimates that a $50 note will be needed to pay for a number of items at a supermarket (Applying Strategies)
• calculate the cost of several items and tender the appropriate amount (Applying Strategies, Reasoning)
• estimate the amount of change due and check using a calculator eg for a purchase of $3.50 if a $5 note is tendered (Applying Strategies, Reasoning)</td></tr>
</table>

Money

NLS.15 A student plans personal finances.

Knowledge and Skills	**Working Mathematically**
Students learn about • identifying financial issues which influence their daily lives • identifying and describing financial terms eg income, expenditure, saving, borrowing, interest • identifying personal funds available for specific purposes • recognising the need to balance expenses with available funds • recognising the relationship between value and price of a range of goods and services • making informed decisions about purchasing goods and services while considering value for money • calculating the actual costs incurred when paying for goods and services with cash, on lay-by, by credit card, on terms, or with loans • calculating the amount of time it will take to pay off items on credit cards or on terms at a specific rate per week or month	**Students learn to** • calculate income available from a range of sources eg allowance, casual or part-time work *(Applying Strategies, Reflecting)* • allocate amounts of money from an allowance for specific purposes eg 'From my $10 allowance I need to keep $5 for entry to the swimming pool, so I have $5 to spend or save' *(Applying Strategies, Reflecting)* • develop a budget to meet personal financial needs *(Applying Strategies, Reflecting)* • use financial services in a variety of ways eg over the counter, ATM, EFTPOS, cheque book, telephone banking, internet banking, credit cards, lay-by, hire purchase *(Applying Strategies, Reflecting)* • keep and check records of financial transactions eg - keep card/PIN number confidential and in a safe place - retain card and receipt after using ATM - retain and check receipts after purchasing goods and services - record receipt number when using telephone or internet services to make payments - retain and review financial statements *(Applying Strategies, Reflecting)* • compare the interest rates and other costs for a range of services from various financial institutions *(Applying Strategies, Reflecting)* • calculate the total cost of purchasing goods using credit cards or on terms and determine a 'best buy' *(Applying Strategies, Reasoning)*

Chance and Probability	
NLS.16 A student recognises and describes the elements of chance in everyday events.	
Knowledge and Skills **Students learn about** • recognising the element of chance in familiar events eg tossing a die, coin • distinguishing between events that are certain to occur and those that may occur eg having a birthday, winning a lottery • describing the likelihood of familiar events eg might, certain, probable, likely, unlikely, possible, impossible • interpreting numerical values assigned to the likelihood of events occurring in real life contexts eg 50:50, 1 in 2, 1 in 100, 1 in a million	**Working Mathematically** **Students learn to** • use the language of chance in everyday situations *(Communicating)* • predict possible outcomes in everyday situations eg deciding what might occur in a movie before the ending of the story *(Reflecting)* • ask questions related to the likelihood of events eg 'Do I need to take my umbrella if the sky is grey?' *(Reflecting)* • evaluate the likelihood of winning prizes in lotteries and other competitions *(Reasoning)*

Patterns and Algebra

Patterns and Algebra	
PALS.1 A student recognises repeating patterns.	
Knowledge and Skills **Students learn about** • recognising what comes next in a repeating pattern using familiar objects eg blue button, red button, blue button, red button, ___________ • recognising what comes next in a simple pattern using concrete shapes eg square, circle, triangle, square, circle, ___________ • recognising what comes next in a simple sound or action pattern eg two short claps, one clap, two short claps, ___________	**Working Mathematically** **Students learn to** • copy a pattern involving familiar objects eg threading beads on a necklace, placement of knives and forks on a dinner table *(Applying Strategies)* • complete a pattern involving familiar objects eg a place setting at a dinner table, put a program on every second chair *(Applying Strategies)* • recognise repeating patterns in the environment eg paving patterns, wallpaper, music *(Reflecting)* • recognise when an error occurs in a pattern and describe what is wrong eg when making a necklace, recognises a red bead has been used instead of a blue, and corrects the error *(Applying Strategies, Reasoning)* • ask questions about how repeating patterns are made and how they can be copied or continued *(Questioning)*
PALS.2 A student recognises and continues number patterns.	
Knowledge and Skills **Students learn about** • creating number patterns using concrete materials eg blocks, shapes • continuing simple number patterns that increase eg 2, 4, 6, 8, ________ • continuing simple number patterns that decrease eg 20, 18, 16, 14, ________ • describing number patterns when counting forwards or backwards eg 3, 6, 9, 12 where three is added each time	**Working Mathematically** **Students learn to** • ask questions about how number patterns are made and how they can be copied and continued *(Questioning)* • describe how the next element in a number pattern was determined *(Communicating, Reasoning)* • represent number patterns using diagrams, words or symbols *(Communicating)*

Patterns and Algebra	
PALS.3 A student calculates missing values by completing simple number sentences.	
Knowledge and Skills	**Working Mathematically**
Students learn about • completing number sentences involving one operation by calculating missing values eg find □ if □ + 5 = 8; find □ if □ × 3 = 12 • using letters (pronumerals) to represent numbers in solving number sentences involving one operation eg $x + 6 = 14$; $18 \div x = 9$	**Students learn to** • describe strategies for calculating missing values *(Communicating)* • use a number sentence to solve a given problem eg 'I have $25 and the CD costs $31. How much more do I need to purchase the CD?' *(Applying Strategies)*

Data

<table>
<tr><th colspan="2">Data</th></tr>
<tr><td colspan="2">DLS.1 A student reads and interprets tables and data displays.</td></tr>
<tr><td>Knowledge and Skills

Students learn about

• reading information from graphs using features such as heading/title of graph, labels on axes, scale, key
• interpreting information presented in tables and graphs to answer questions eg ‘The columns show that there are more boys than girls’</td><td>Working Mathematically

Students learn to

• read graphs and tables from a variety of sources (Communicating, Applying Strategies)
• interpret graphs and tables from a variety of sources eg self-made, other students, books, newspapers, other KLAs, internet, factual texts (Communicating, Applying Strategies)
• compare tables and graphs constructed from the same data to determine which is the most appropriate method of display (Reasoning)
• draw conclusions on the basis of the information displayed in tables and graphs (Reasoning)</td></tr>
<tr><td colspan="2">DLS.2 A student gathers, organises and displays data.</td></tr>
<tr><td>Knowledge and Skills

Students learn about

• collecting data about themselves and their environment
• sorting collected data into groups
• keeping track of what has been counted by using concrete materials, tally marks, words or symbols
• using objects or pictures to represent data, using one-to-one correspondence eg using a block to represent each car
• creating simple tables to organise data
• displaying data using picture, column and line graphs
• following conventions for displaying data including equal spacing, same-sized symbols, key for symbols, headings, labelling axes</td><td>Working Mathematically

Students learn to

• pose questions that can be answered by gathering and displaying data (Questioning)
• pose a question to be answered using a survey eg ‘What is the most popular sport among students in our class?’ (Questioning)
• determine what data to gather to investigate a question (Reasoning)
• predict the likely results of a student-designed survey (Reflecting)
• create a table to organise collected data, using a computer program eg spreadsheet (Applying Strategies)
• use computer software to draw graphs eg picture graphs, column graphs, bar graphs, sector graphs (Applying Strategies)
• interpret graphs to answer a survey question and/or draw conclusions eg ‘Swimming is the most popular sport among students in our class’ (Applying Strategies, Reasoning)
• identify misleading representations of data eg where the symbols are not the same size (Applying Strategies)</td></tr>
</table>

Measurement

Time	
MLS.1 A student matches familiar activities with time frames.	
Knowledge and Skills **Students learn about** • associating familiar activities involving eating, personal care and social routines with times of the day	**Working Mathematically** **Students learn to** • indicate an association (using personalised strategies) between a time of the day and a range of familiar activities eg breakfast is in the morning, dinner is in the evening, changing for bed in the evening, dressing for school in the morning *(Applying Strategies, Reflecting)* • recognise activities that occur on weekdays eg what happens at school on Mondays *(Applying Strategies, Reflecting)* • recognise activities that occur on the weekend eg sport, outings *(Applying Strategies, Reflecting)* • identify activities that occur on specific days and at specific times eg gym group is on Wednesday evenings during school terms, the disco is held every second Saturday in the afternoon *(Applying Strategies, Reflecting)*
MLS.2 A student recognises and uses the language of time.	
Knowledge and Skills **Students learn about** • recognising the language of time in relation to specific personal activities • recognising the language of time in a range of everyday situations • using the language of time to describe activities in a range of everyday situations	**Working Mathematically** **Students learn to** • use or respond to the language of time in relation to a range of personal activities eg 'It is now 12 o'clock and it's time for lunch', 'It is time to pack up because the bus will be here in 10 minutes' *(Communicating)* • respond to questions involving the language of time eg 'Did you have your shower before or after breakfast?', 'Will you be going shopping this afternoon?' *(Communicating)* • use the language of time to describe personal activities and events eg 'I did my homework after dinner last night', 'I will be going to the football tomorrow afternoon', 'There was a delay of half an hour this morning on the school bus', 'I will be going to a barbecue next weekend', 'The holidays are only three weeks away' *(Communicating)*

<table>
<tr><td colspan="2">Time</td></tr>
<tr><td colspan="2">MLS.3 A student reads and interprets time in a variety of situations.</td></tr>
<tr><td>Knowledge and Skills

Students learn about

Clocks and Watches
• reading the hour on digital clocks or watches eg ‘It’s 11 o’clock’
• reading the hour on analog clocks or watches eg ‘It’s 4 o’clock’
• reading half hour and quarter hour on digital clocks or watches eg ‘It’s half past 4’, ‘It’s 4:15’
• reading half hour and quarter hour on analog clocks or watches eg ‘It’s a quarter to 5’
• reading minutes on clocks or watches eg ‘It’s 6 minutes past 1’, ‘It’s 12 minutes to 6’
• describing the relationship between analog and digital time eg ‘12:30 is the same as half past 12’
• reading am and pm on digital clocks or watches

Calendars and Planners
• reading the names or symbols for days of the week on a calendar
• reading the months of the year on a calendar
• locating special days and events on a calendar eg ‘Anzac Day is on the 25th of April’
• recognising that calendars are used to plan special events and activities eg school term plan in the newsletter, coming events in the newspaper
• identifying number of days, weeks, months, between one event and another eg ‘It’s three days until the weekend’, ‘The term has four more weeks’</td>
<td>Working Mathematically

Students learn to

Clocks and Watches
• use ‘hour’ within a personal context eg ‘1 o’clock means it’s time for lunch’ (Reflecting)
• use ‘half hour’ within a personal context eg ‘Half past 3 means it’s time to leave’ (Reflecting)
• use minutes within a personal context eg ‘I need to catch the bus at 13 minutes past 5’ (Reflecting)
• respond to questions related to time eg ‘What time does your bus leave?’ (Communicating)
• ask questions related to time eg ‘How long until the bell?’ (Questioning)

Timetables
• read and follow an individual sequence chart (timetable) for a range of activities eg reads the classroom timetable to find out if a particular activity (eg sport) happens on that day (Applying Strategies, Reflecting)
• read and follow a school timetable for group or class activities eg follows a class timetable to work out which lesson is next (Applying Strategies, Reflecting)
• read and interpret a written timetable for TV programs eg works out when a particular TV show is on (Applying Strategies, Reflecting)
• read and interpret a timetable for using community transport eg read a bus timetable at a bus stop to work out when the next bus is due (Applying Strategies, Reflecting)

Calendars and Planners
• locate birthdays of significant people on a calendar eg family, friends (Reflecting)
• use a calendar/diary to plan for regular personal activities eg swimming every second Friday, PE each Tuesday (Applying Strategies, Reflecting)
• use a calendar to plan special events and activities eg camp, birthday party (Reflecting)
• use a calendar or planner to calculate time for particular activities eg blocks out three weeks for completion of a school project (Reflecting)
• use electronic formats of calendars and planners (Applying Strategies)</td></tr>
</table>

<table>
<tr><td colspan="2">Time</td></tr>
<tr><td colspan="2">MLS.4 A student organises personal time and manages scheduled activities.</td></tr>
<tr><td>Knowledge and Skills

Students learn about

• identifying the amount of time needed for a range of activities eg ‘I need half an hour to have a shower and get dressed’, ‘It takes me 10 minutes to walk from home to the railway station’
• structuring activities of a school day in relation to the time required for each event eg ‘Get up at 7 am, prepare for school, shower, dress and have breakfast. Leave home at 8 am. Lessons from 9 am to 3 pm. Attend youth group from 4 pm to 5 pm. Catch bus home at 5:15 pm’
• making choices and decisions about activities on the basis of time available eg ‘I will need to visit my friend on Tuesday because on Wednesday afternoon I have a regular youth group activity’, ‘I can’t attend the barbecue until 1:30 pm because I have an appointment to have a haircut at 1 o’clock’
• planning personal time over a day or a week so that activities do not clash
• scheduling events over a day or week taking into account a range of activities and personal responsibilities</td><td>Working Mathematically

Students learn to

• recognise that specific activities require a particular amount of time (Reflecting)
• recognise the order and sequence of events in relation to carrying out regular routines (Reflecting)
• identify priorities in relation to personal time, and discriminate between essential and non-essential activities (Reasoning)
• prepare a personal timetable for a school day which includes all scheduled activities (Applying Strategies, Reflecting)
• prepare a personal timetable for a weekend (Applying Strategies, Reflecting)</td></tr>
</table>

<table>
<tr><td colspan="2">Measurement</td></tr>
<tr><td colspan="2">MLS.5 A student responds to the language of measurement in everyday contexts.</td></tr>
<tr><td>Knowledge and Skills

Students learn about

• responding to language related to length, height and distance eg long, short, tall, higher than, lower than, the same as, near, far, closer
• responding to language related to temperature eg hot, cold, warm, lukewarm, freezing, boiling, hotter, colder
• responding to language related to the mass of objects eg light, heavy, harder to push/pull, heavier, lightest
• responding to language related to perimeter and area eg bigger/smaller than, the same as, surface, outside, inside, distance around
• responding to language related to capacity and volume eg full, empty, half-full, has more/less, will hold more/less</td><td>Working Mathematically

Students learn to

• make comparisons based on attributes that can be measured eg ‘This box is heaviest’, ‘John is the tallest boy in the class’ (Applying Strategies)
• respond to instructions that involve language of comparison ‘Please bring me the empty container’, ‘Put the heavy book on the bottom shelf’ (Applying Strategies)</td></tr>
</table>

Measurement	
MLS.6 A student uses the language of measurement in everyday contexts.	
Knowledge and Skills **Students learn about** • using language to describe length, height and distance eg long, short, tall, higher than, lower than, the same as, near, far, closer • using language to describe temperature eg hot, cold, warm, lukewarm, freezing, boiling, hotter, colder • using language to describe the mass of objects eg light, heavy, harder to push/pull, heavier, lightest • using language to describe perimeter and area eg bigger/smaller than, the same as, surface, outside, inside, distance around • using language to describe capacity and volume eg full, empty, half-full, has more/less, will hold more/less	**Working Mathematically** **Students learn to** • make comparisons based on attributes that can be measured eg 'I am taller than my brother' *(Applying Strategies, Communicating)* • compare objects using attributes that can be measured eg 'This box is the heaviest' *(Applying Strategies)* • ask questions that can be answered using attributes that can be measured eg 'Which day was the coldest according to this graph?' *(Questioning)*
MLS.7 A student estimates and measures temperature.	
Knowledge and Skills **Students learn about** • recognising informal ways in which temperature can be estimated eg steam from a kettle indicates boiling water, colour indicators for hot and cold water taps • recognising formal ways in which temperature is measured eg digital thermometers, oven dials • using formal units to measure temperature eg degrees • using abbreviations for units of measurement related to temperature eg ^{o}C	**Working Mathematically** **Students learn to** • predict whether an item of food will be hot and explain their prediction *(Reasoning, Reflecting)* • describe how the temperature of an object was measured *(Communicating)* • set the oven temperature according to a recipe *(Applying Strategies)* • solve everyday problems related to temperature eg choose clothing appropriate for the weather conditions *(Applying Strategies)*

Measurement

MLS.8 A student estimates and measures length and distance.

Knowledge and Skills	Working Mathematically
Students learn about • using informal ways in which length and distance can be estimated eg looking at which toilet block is closer, placing objects side by side to determine which is longer • using instruments to measure length and distance eg ruler, tape measure, trundle wheel, odometer • using formal units to measure length and distance eg millimetres, centimetres, metres, kilometres • reading and using abbreviations for units of measurement eg mm, cm, m, km • using the relationship between units of length eg 10 mm = 1 cm, 100 cm = 1 metre • reading scales on maps and diagrams • using the term 'perimeter' to describe the total distance around a shape • estimating and measuring perimeter eg distance around the school oval	**Students learn to** • predict whether an object will be longer or shorter than another object and explain their prediction *(Reasoning, Reflecting)* • select and use the appropriate instruments to measure lengths and distances eg tape measure, ruler, trundle wheel *(Applying Strategies)* • describe how a length or distance was measured *(Communicating)* • interpret scales on maps to calculate distances *(Applying Strategies, Communicating)* • solve everyday problems related to length and distance eg plant seedlings 20 cm apart *(Applying Strategies)*

MLS.9 A student estimates and measures capacity.

Knowledge and Skills	Working Mathematically
Students learn about • estimating capacity in informal ways eg determining if the contents of one container fit into another, observing the amount of space an item requires compared to another • using formal units to measure capacity eg litres, millilitres • using calibrated measuring devices eg measuring cups and jugs, medicine glasses, beakers • reading and using abbreviations for units of capacity eg L, mL • recognising the relationship between units of capacity eg 1000 mL = 1 L	**Students learn to** • predict whether a container will hold more or less than another container and explain their prediction *(Reflecting, Reasoning)* • select and use the appropriate formal devices to measure capacity eg use a medicine glass to measure medicine rather than a jug *(Applying Strategies)* • describe how the capacity of an object was measured *(Communicating)* • solve everyday problems related to capacity eg using half a litre of milk in a recipe for pancakes *(Applying Strategies)*

Measurement	
MLS.10 A student estimates and measures mass.	
Knowledge and Skills **Students learn about** • estimating and comparing masses informally eg 'This book feels heavier than this one' • using measuring devices to measure mass eg kitchen scales, bathroom scales, weigh stations • using formal units to measure mass eg gram, kilogram • reading and using abbreviations for units of mass eg g, kg, t • recognising the relationship between commonly used units of mass eg 1000 g = 1 kg, 500 g = $\frac{1}{2}$ kg	**Working Mathematically** **Students learn to** • ask questions about why an item can or cannot be lifted *(Questioning)* • predict which item would be heavier than, lighter than or about the same mass as another item and explain their prediction *(Reasoning)* • select and use appropriate measuring devices to measure mass eg kitchen scales to weigh flour *(Applying Strategies)* • describe how the mass of an object was measured *(Communicating)* • solve everyday problems related to mass *(Applying Strategies)*
MLS.11 A student estimates and measures area.	
Knowledge and Skills **Students learn about** • measuring area in informal ways eg 'I need six sheets of paper to cover the table' • comparing areas informally eg 'The table cloth is larger than the surface of the table' • measuring areas in square centimetres using a grid overlay • measuring areas in square metres • reading and using abbreviations for units of area	**Working Mathematically** **Students learn to** • explain where square metres are used for measuring in everyday situations eg floor coverings *(Applying Strategies, Communicating)* • describe how the area of an object was measured *(Communicating)* • solve everyday problems related to area *(Applying Strategies)*

Space and Geometry

Three-dimensional and two-dimensional space

SGLS.1 A student matches and sorts three-dimensional objects.

Knowledge and Skills	Working Mathematically
Students learn about • matching three-dimensional objects based on an attribute eg shape, size, function • sorting three-dimensional objects based on an attribute eg shape, colour, size, function • recognising three-dimensional objects from different orientations	**Students learn to** • sort items on the basis of their shape, size, function eg crockery, cutlery, sports equipment, clothes for washing *(Applying Strategies)* • stack items on the basis of their shape eg crockery, chairs, books *(Applying Strategies)* • indicate the reasons for sorting items in a particular way *(Communicating, Reasoning)* • predict the ways in which particular items can be stacked *(Applying Strategies)*

SGLS.2 A student matches and sorts two-dimensional shapes.

Knowledge and Skills	Working Mathematically
Students learn about • matching, sorting and identifying two-dimensional shapes	**Students learn to** • sort two-dimensional shapes according to features such as size and shape *(Applying Strategies)* • identify and match circles, squares, triangles and rectangles *(Applying Strategies)* • sort two-dimensional shapes on the basis of a given attribute eg number of corners or sides *(Applying Strategies)* • match and sort two-dimensional shapes when presented in different orientations *(Applying Strategies, Reasoning)*

Three-dimensional and two-dimensional space	
SGLS.3 A student identifies the features of three-dimensional objects and two-dimensional shapes.	
Knowledge and Skills	**Working Mathematically**
Students learn about • recognising the attributes of a variety of three-dimensional objects in a range of contexts • recognising and describing the attributes of two-dimensional shapes • visualising the resulting shape when a three-dimensional object is cut in half eg an orange • visualising the result of putting together (or separating) two-dimensional shapes eg 'This house shape is made up of a triangle and a square'	**Students learn to** • identify and name three-dimensional objects that are used in everyday situations eg cones, cubes and cylinders *(Applying Strategies)* • identify three-dimensional objects in pictures, computer displays and within the environment *(Applying Strategies)* • identify circles, squares, triangles and rectangles in the built environment *(Applying Strategies, Reflecting)* • construct and describe models using a variety of three-dimensional objects *(Applying Strategies)* • draw two-dimensional shapes using computer software *(Applying Strategies)*

Position	
SGLS.4 A student responds to the language of position.	
Knowledge and Skills	**Working Mathematically**
Students learn about • recognising and responding to the language of position in a range of contexts	**Students learn to** • identify preference for a position in response to a question eg 'Would you rather lie on your side or sit in the chair?', 'Would you rather sit next to John or Sam?' *(Communicating)* • follow spoken instructions relating to the language of position eg 'Put your raincoat on the top hook', 'Take the books from the cabinet behind the desks', 'Please move inside the carriage' *(Communicating)* • follow symbols and written instructions relating to the language of position eg follow arrows to locate an office on an upper floor, follow symbols to carry a container right side up, follow written instructions to stack items in a storeroom *(Applying Strategies)*

Position

SGLS.5 A student uses the language of position in a variety of situations.

Knowledge and Skills	Working Mathematically
Students learn about	**Students learn to**
• using the language of position	• indicate the position of objects or buildings in response to a question eg 'The books are on the shelf in the classroom', 'The seats are under the trees in the playground', 'The supermarket is next to the garage in the main street', 'The yellow tulips are in the middle of the row in the garden' *(Applying Strategies)* • describe the position of objects or buildings in a range of contexts eg 'I went to the ticket office inside the railway station', 'The bus stop is opposite the main gate', 'Appliances are located on the ground floor', 'This lift goes to the upper level', 'Tickets are purchased at the office beside the turnstiles' *(Communicating, Applying Strategies)* • give directions in a range of contexts eg 'Stand behind the line to throw the ball', 'Walk towards the doorway', 'Turn left at the top of the stairs' *(Communicating, Applying Strategies)*

SGLS.6 A student uses simple maps and plans.

Knowledge and Skills	Working Mathematically
Students learn about	**Students learn to**
• recognising the purpose and functions of maps and plans • using simple maps and plans to locate position and follow routes • locating specific sites using grid references in street directories and road maps	• locate their seat on a plan of the classroom *(Applying Strategies)* • locate their classroom on a plan of the school *(Applying Strategies)* • draw a plan of the classroom showing key features such as doors, windows, tables, chairs, cupboards *(Applying Strategies)* • draw a plan of the school showing classroom, playground and administration areas *(Applying Strategies)* • use a map to show direction from home classroom to the library (*Applying Strategies*) • use a map to show location of their home *(Applying Strategies)* • use maps for a variety of purposes eg use a street directory to locate a venue in the community *(Applying Strategies)*

11 Assessment

11.1 Standards

The Board of Studies K–10 curriculum framework is a standards-referenced framework that describes, through syllabuses and other documents, the expected learning outcomes for students.

Standards in the framework consist of two interrelated elements:

- outcomes and content in syllabuses showing what is to be learned
- descriptions of levels of achievement of that learning.

Exemplar tasks and student work samples help to elaborate standards.

Syllabus outcomes in Mathematics contribute to a developmental sequence in which students are challenged to acquire new knowledge, skills and understanding.

The standards are typically written for two years of schooling and set high, but realistic, expectations of the quality of learning to be achieved by most students by the end of Years 2, 4, 6, 8, 10 and 12.

Using standards to improve learning

Teachers will be able to use standards in Mathematics as a reference point for planning teaching and learning programs, and for assessing and reporting student progress. Standards in Mathematics will help teachers and students to set targets, monitor achievement, and as a result make changes to programs and strategies to support and improve each student's progress.

11.2 Assessment for learning

Assessment for learning in Mathematics is designed to enhance teaching and improve learning. It is assessment that gives students opportunities to produce the work that leads to development of their knowledge, skills and understanding. Assessment for learning involves teachers in deciding how and when to assess student achievement, as they plan the work students will do, using a range of appropriate assessment strategies including self-assessment and peer assessment.

Teachers of Mathematics will provide students with opportunities in the context of everyday classroom activities, as well as planned assessment events, to demonstrate their learning.

In summary, assessment for learning:

- is an essential and integrated part of teaching and learning
- reflects a belief that all students can improve
- involves setting learning goals with students
- helps students know and recognise the standards they are aiming for
- involves students in self-assessment and peer assessment
- provides feedback that helps students understand the next steps in learning and plan how to achieve them
- involves teachers, students and parents reflecting on assessment data.

Quality Assessment Practices

The following principles provide the criteria for judging the quality of assessment materials and practices.

Assessment for learning:

- **emphasises the interactions between learning and manageable assessment strategies that promote learning**
  In practice, this means:
  - teachers reflect on the purposes of assessment and on their assessment strategies
  - assessment activities allow for demonstration of learning outcomes
  - assessment is embedded in learning activities and informs the planning of future learning activities
  - teachers use assessment to identify what a student can already do

- **clearly expresses for the student and teacher the goals of the learning activity**
  In practice, this means:
  - students understand the learning goals and the criteria that will be applied to judge the quality of their achievement
  - students receive feedback that helps them make further progress

- **reflects a view of learning in which assessment helps students learn better, rather than just achieve a better mark**
  In practice, this means:
  - teachers use tasks that assess, and therefore encourage, deeper learning
  - feedback is given in a way that motivates the learner and helps students to understand that mistakes are a part of learning and can lead to improvement
  - assessment is an integral component of the teaching-learning process rather than being a separate activity

- **provides ways for students to use feedback from assessment**
  In practice, this means:
  - feedback is directed to the achievement of standards and away from comparisons with peers
  - feedback is clear and constructive about strengths and weaknesses
  - feedback is individualised and linked to opportunities for improvement

- **helps students take responsibility for their own learning**
  In practice, this means:
  - assessment includes strategies for self-assessment and peer assessment emphasising the next steps needed for further learning

- **is inclusive of all learners**
  In practice, this means:
  - assessment against standards provides opportunities for all learners to achieve their best
  - assessment activities are free of bias.

Making judgements about student achievement

Assessment for learning in the *Mathematics Years 7–10 Syllabus* is designed to give students opportunities to produce the work that leads to development of their knowledge, skills and understanding. It involves teachers in deciding how and when to assess student achievement, as they plan the work students will do, using a range of appropriate assessment strategies including self-assessment and peer-assessment. Teachers of Years 7–10 Mathematics provide students with opportunities in the context of everyday classroom activities, as well as planned assessment events, to demonstrate their learning.

Gathered evidence can also be used for *assessment of learning* that takes place at key points in the learning cycle, such as the end of a Year or Stage, when schools may wish to report differentially on the levels of skill, knowledge and understanding achieved by students. Descriptions of levels of achievement for Stage 4 and Stage 5 in Mathematics have been developed to provide schools with a useful reporting tool that they can use to report consistent information about student achievement to students and parents, and to the next teacher, to help to plan the next steps in the learning process.

11.3 Reporting

Reporting is the process of providing feedback to students, parents and other teachers about students' progress.

Levels of achievement have been written for Mathematics Years 7–10. These describe observable and measurable features of student achievement at the end of a Stage within the indicative hours of study.

Levels of achievement will provide a common language for reporting. This can make it easier for students, parents and teachers to understand how a student is progressing and to set expectations for the next steps in learning.

At Stage 4, teachers of Mathematics Years 7–10 will be able to use the descriptions of levels of achievement to make an on-balance judgement, based on the available assessment evidence, to match each student's achievement to a level description. The descriptions will help identify those students who have a solid foundation for the next Stage of learning, as well as those that have not met the expected standard by the end of the Stage and who may require additional time or different strategies to consolidate their learning.

At the end of Year 10, teachers of Mathematics Years 7–10 will make an on-balance judgement, based on the available assessment evidence, to match each student's achievement to a level description. This level will be reported on the student's School Certificate Record of Achievement.

11.4 Choosing Assessment Strategies

In Years 7–10 Mathematics, assessment of student learning should incorporate measures of students':
- ability to work mathematically
- knowledge, understanding and skills related to: Number; Patterns and Algebra; Data; Measurement; and Space and Geometry.

Students indicate their level of understanding and skill development in what they do, what they say, and what they write and draw. The most appropriate method or procedure for gathering assessment information is best decided by considering the purpose for which the information will be used, and the kind of performance that will provide the information. Consequently there is a variety of ways to gather assessment information in Mathematics. Tasks given to students for the purpose of gathering assessment information include projects, investigations, oral reports or explanations, tests and practical assignments. For example, practical tasks would often be an appropriate strategy for the assessment of achievement of outcomes for Measurement.

Teachers have the opportunity to observe and record aspects of students' learning in a range of situations. When students are working in groups, teachers are well placed to determine the extent of student interaction and participation. By listening to what students say – including their responses to questions or other input – teachers are able to collect many clues about students' existing understanding. Through interviews (which may only be a few minutes in duration), teachers can collect specific information about the ways in which students think in certain situations. The students' responses to questions and comments will often reveal their levels of understanding, interests and attitudes. Records of such observations form valuable additions to information gained using other assessment strategies, and enhance teachers' judgement of their students' achievement of outcomes. Consideration of students' journals or their comments on the process of gaining a solution to a problem can also provide valuable insight into the extent of students' mathematical thinking.

Possible sources of information for assessment purposes include the following:
- samples of students' work
- explanation and demonstration to others
- questions posed by students
- student-produced overviews or summaries of topics
- practical tasks such as measurement activities
- investigations and/or projects
- students' oral and written reports
- short quizzes
- pen-and-paper tests involving multiple choice, short-answer questions and questions requiring longer responses, including interdependent questions (where one part depends on the answer obtained in the preceding part)
- open-book tests
- comprehension and interpretation exercises
- student-produced worked examples
- teacher/student discussion or interviews
- observation of students during learning activities, including listening to students' use of language
- observation of students' participation in a group activity
- consideration of students' portfolios
- students' plans for and records of their solutions of problems
- students' journals and comments on the process of their solutions.